The Real Effects of Financing and Trading Frictions*

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Abstract

I develop a model revealing the interplay between a stock's liquidity and the policies and value of the issuing firm. The model shows that bid-ask spreads increase not only the firm's cost of capital but also the opportunity cost of cash, then lowering a firm's cash reserves, increasing its liquidation risk, and reducing firm value. These outcomes are stronger when internalized by liquidity providers, simultaneously leading to a wider bid-ask spread. A two-way relation between the firm and the bid-ask spread arises, implying that shocks arising within the firm or in the stock market have more complex implications than previously understood.

Keywords: Financial constraints, transaction costs, real effects of financial markets, small firms

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1 Introduction

Corporate financial constraints and investors' trading frictions appear to go hand in hand in the cross-section of firms. Large firms enjoy an easy access to external financing, and their stocks are very liquid. At the other side of the spectrum, small firms are largely financially constrained, and their stocks are relatively illiquid. Indeed, small firms typically face delays and costs when raising fresh funds, an issue that has spurred the creation of an ad-hoc committee within the U.S. Securities and Exchange Commission (SEC).[1] Moreover, their stocks are characterized by non-negligible bid-ask spreads, low trading volume, and other microstructure frictions (e.g., Novy-Marx and Velikov, 2016; Chordia et al., 2011). As small firms represent more than 80% of U.S. firms over the past forty years (Hou et al., 2020), understanding the frictions affecting their performance is of utmost importance.

This paper develops a dynamic model showing that trading frictions and financial constraints are deeply related and investigates their real effects. The model studies a firm that has assets in place—which generate a stochastic flow of revenues—and a growth option. The firm faces uncertainty in its ability to raise external financing, as small firms do in the real world. Crucially, the firm's shareholders face frictions when trading the stocks, as is typically the case for small-capitalization firms. The model shows that the bid-ask spread borne by firm shareholders affect corporate policies and value. In turn, corporate policies and value feed back into the bid-ask spread. Thus, the model highlights a two-way relation between the policies and value of the firm and the liquidity of its stock. Through this relation, shocks arising within the firm or in financial markets end up bearing a more nuanced impact than understood by previous works.

To understand the strengths at play, consider first how a positive bid-ask spread affects the optimal policies and value of the issuing firm. By leading to a loss when trading the stock, a bid-ask spread leads shareholders to require a larger return to invest in the stock.

[1]Small firms are typically less known and more vulnerable to capital market imperfections. In contrast, large, established firms are more likely to have continued relations with financial institution. The U.S. SEC Advisory Committee in Small and Emerging Companies pointed out that small firms often struggle to attract capital, see https://www.sec.gov/spotlight/advisory-committee-on-small-and-emerging-companies.shtml.

I.e., in the spirit of Amihud and Mendelson (1986), the cost of capital increases. At the same time, the greater cost of capital also implies an increase in the opportunity cost of keeping cash inside the firm. On net, I demonstrate that firms whose stocks are traded at larger bid-ask spreads are more financially constrained. These firms are less likely to raise external financing and keep smaller precautionary cash reserves and, thus, are more exposed to forced liquidations. Moreover, these firms face a severe underinvestment problem, as the additional return required by the investors erodes the profitability of investment opportunities. Overall, a positive bid-ask spread leads to a decrease in firm value.

When liquidity providers internalize such illiquidity-driven drop in firm value, a feedback effect arises. Realistically, the model acknowledges that liquidity providers may be distracted by trading opportunities in other markets. When liquidity provision is competitive, the equilibrium bid-ask spread then makes liquidity providers indifferent about making the market for the stock or pursuing such outside opportunities. In this richer setup, the illiquidity-driven decrease in firm value leads to an increase in the bid-ask spread that makes their indifference condition binding. As a result, the detrimental effects of stock illiquidity on corporate policies and value strengthen—in particular, the firm holds less cash, financial constraints tighten, the probability of liquidations increases, and firm value declines. The ensuing two-way relation between the bid-ask spread and the policies and value of the issuing firm implies that shocks affecting liquidity provision also impact corporate outcomes. Similarly, shocks affecting firm operations also impact the liquidity of its stock.

The model delivers a rich set of predictions. First of all, it provides a unified framework that supports the empirical evidence on the impact of bid-ask spreads on corporate outcomes. The model predicts that these firms should face severe financial constraints because of their larger costs of external *and* internal equity, which reduce the probability of external financing and the size of the firm's precautionary cash reserves (consistent with Nyborg and Wang (2021)). As a result, these firms face higher liquidation risk, consistent with Brogaard et al. (2017). Notwithstanding these constraints, these firms should exhibit larger payouts in the cross-section to compensate investors for trading frictions, as documented by Banerjee et al.

(2007). These firms should also suffer from underinvestment, consistent with Campello et al. (2014) and Amihud and Levi (2022).

Moreover, harnessing the interplay between the firm and the liquidity of its stock, the model delivers novel testable predictions. It shows that shocks hampering liquidity provision in the market of the stock have an amplified effect on the bid-ask spread when reflected into firm value. As a direct effect, such shocks naturally raise the competitive bid-ask spread. Furthermore, as illustrated, the ensuing greater costs of trading borne by the firm's shareholders affect corporate policies, tighten financial constraints, and decrease firm value, widening the bid-ask spread further. Through this mechanism, shocks originating in the market of the firm's stock propagate to firm policies and outcomes, eventually bearing an amplified impact on the liquidity of the firm stock.

The interplay between the firm and the liquidity of its stock also implies that shocks to firm attributes bear novel indirect effects, which have been overlooked by the existing literature. The analysis reveals that a tightening in the firm's access to external financing or an increase in its cash flow volatility—which the literature recognizes as prime drivers of corporate cash reserves (see, e.g. Opler et al., 1999; Bates et al., 2009)—affect a firm's cash hoarding behavior not only by increasing its precautionary demand (as highlighted by previous works) but also by increasing its opportunity cost. This latter channel is novel and arises as such shocks affect the firm's bid-ask spread too. Importantly, this channel hampers the firm's ability to accumulate cash exactly when its demand for cash increases—i.e., it makes cash costlier when the firm needs it the most.[2] Thus, revisiting empirical tests to account for stock liquidity would shed new light on the quantitative impact of such standard cash determinants.

Relatedly, and notably, this novel channel exacerbates the impact of adverse shocks on the firm's probability of liquidation. Indeed, a deterioration in the firm's access to external financing or an increase in cash flow volatility both raise the probability of liquidation of constrained firms. Yet, as such shocks affect stock liquidity too (as just described), they also

[2] I.e., when external financing tightens or when cash flows are more volatile.

make precautionary cash hoarding more costly and decrease the firm's target cash level. This additional effect makes firms less financially resilient. I.e., firms are less able to withstand prolonged periods of losses and, after a given cumulative loss, they exhibit a higher probability of forced liquidation. Overall, the analysis then indicates that measures of financial constraints could fruitfully account for stock illiquidity to improve their predictive power.

The model predictions are robust to alternative modeling assumptions, as shown in the Supplementary Appendix. First, I acknowledge that the market of more illiquid stocks may assume an over-the-counter (OTC) market structure, whereby shareholders may not be able to find liquidity providers at times of need and, if they do, they need to bargain over the terms of the trade. I show that the results are robust to this alternative modeling. In addition, given the focus on firm's financial policies, it is crucial that the results are not driven by specific assumptions. I show that the effects of stock illiquidity on corporate choices are robust to modeling financing frictions as issuance costs (instead of capital supply uncertainty) as well as when allowing the firm to borrow from bank credit lines as an additional source of liquidity.

The model provides a tractable framework to study regulation targeting equity markets from a corporate perspective. In particular, academics and policy-makers have been questioning whether market forces guarantee enough liquidity provision, especially for small-capitalization stocks. Some stock markets have then contemplated the possibility for firms to engage a designated market maker (DMM) to maintain the bid-ask spread below an agreed-upon level.[3] DMM contracts can help decrease the firm's cost of capital by capping investors' trading costs, but bind the firm to correspond a fee to the DMM. On net, the model suggests that small, financially constrained firms would hardly find this contractual agreement value-enhancing if it requires the firm to correspond a periodic fee to the DMM. A contract contemplating a one-off fee might make the DMM service more attractive, though it may not be accessible for firms with low cash reserves or with a tight access to external financing.

[3] See, e.g., Venkataraman and Waisburd (2007), Anand et al. (2009), Menkveld and Wang (2013), or Bessembinder et al. (2015) for papers studying the impact of DMM.

Related literature This paper relates to the literature showing that stock illiquidity impacts corporate policies and outcomes. Fang et al. (2009) show that firms with liquid stocks are more valuable. Campello et al. (2014) find that stock liquidity improves corporate investment and value. Amihud and Levi (2022) show that illiquidity lowers corporate investment, R&D, and inventory. Nyborg and Wang (2021) show that stock liquidity increases a firm's propensity to hold cash. Banerjee et al. (2007) reveal that firms with illiquid stocks pay out more dividends. Brogaard et al. (2017) find that stock liquidity reduces firms' bankruptcy risk. This paper develops a model that rationalizes these empirical findings into a unified framework. It also shows that these effects are amplified when liquidity providers internalize the negative effect of bid-ask spreads on firm value.

The paper is also related to the theoretical literature modeling endogenous feedback effects. Brunnermeier and Pedersen (2009) show that there is a two-way link between an asset's market liquidity and traders' funding liquidity. Traders provide market liquidity, which in turn depends on their funding ability. Because of margin requirements, traders' funding depends on the asset's market liquidity. The current paper instead focuses on the relation between the funding liquidity of a given firm and the market liquidity of its stocks, unraveling a novel two-way relation between a stock's bid-ask spread and the policies and value of the issuing firm. Another related paper is He and Milbradt (2014), who endogenize *bond* illiquidity into a Leland-type model of endogenous default, in which shareholders can inject fresh equity at no cost. He and Milbradt provide a decomposition of credit spreads into a default and a liquidity component, then matching several cross-sectional patterns of bid-ask spreads and credit spreads. Conversely, the present paper focuses on *stock* illiquidity and builds on the strand of dynamic corporate finance models with financing frictions, in which shareholders face costs or uncertainty in their ability to raise additional financing. This paper can reproduce and explain several documented effects of stock illiquidity on real and financial corporate policies (as illustrated above) and provides novel testable implications stemming from the intertwined relation between bid-ask spread and firm value and policies.

Finally, this paper contributes to the strand of dynamic corporate finance models with financing frictions, including Bolton et al. (2011, henceforth BCW); Décamps et al. (2011, henceforth DMRV); Hugonnier et al. (2015, henceforth HMM); Malamud and Zucchi (2019); or Della Seta et al. (2020). These papers show that financing frictions, such as costs or uncertainty in raising external funds, should increase a firm's propensity to keep precautionary reserves. While these extant papers impose an exogenous cost of holding cash, the current model shows that this cost can arise endogenously when accounting for trading frictions faced by firm shareholders. Notably, it illustrates that trading frictions impact both the cost of internal and external financing, then affecting corporate policies and value.

The paper proceeds as follows. Section 2 describes the model. Section 3 describes the model solution. Section 4 derives the model predictions, starting with the corner case in which liquidity provision is exogenous and then moving to the case with endogenous liquidity provision. Section 5 concludes. Proofs are gathered in the Appendix.

2 The model

Time is continuous, and uncertainty is modeled by a probability space $(\Omega, \mathcal{F}, P)$ equipped with a filtration $(\mathcal{F}_t)_{t\geq 0}$. Agents are risk-neutral and discount cash flows at rate $\rho > 0$.

The firm I consider a small firm operating a set of assets in place, which generate a continuous and stochastic flow of cash flows. This flow is modeled as an arithmetic Brownian motion, $(Y_t)_{t\geq 0}$, whose dynamics evolve as

$$dY_t = \mu dt + \sigma dZ_t. \tag{1}$$

The parameters μ and σ are strictly positive and represent the mean and volatility of corporate cash flows, and $(Z_t)_{t\geq 0}$ is a standard Brownian motion. The firm has access to a growth option that has the potential to increase its income stream from dY_t to $dY_t^+ = dY_t + (\mu_+ - \mu)dt$, $\mu_+ > \mu$, by paying a lump-sum cost $I > 0$. That is, the drift can assume two values $\mu_i = \{\mu, \mu_+\}$. Investment is assumed to be irreversible.

The process in equation (1) implies that the firm can make operating losses. If capital supply was perfectly elastic, such losses could be covered by raising outside financing immediately and at no cost. In practice, small firms face financing frictions, such as uncertainty or costs in raising funds. I model this uncertainty by assuming that the firm raises new funds at the jump times of a Poisson process, $(N_t)_{t\geq 0}$, with intensity λ, as in HMM. That is, if the firm decides to raise outside funds, the expected financing lag is $1/\lambda$ periods. If $\lambda \to 0$, the firm cannot raise external funds at all (equivalently, it takes an infinite waiting period to raise fresh funds upon searching) and relies on cash reserves to cover operating losses. If $\lambda \to \infty$, the waiting time upon searching for external funds is zero—i.e., the firm has access to outside financing at no delays.[4] Notably, as shown in the paper, the discount on newly-issued equity is related to trading frictions faced by firms' investors.

Because capital supply is uncertain, the firm has incentives to retain earnings in cash reserves. I denote by $(C_t)_{t\geq 0}$ the firm's cash reserves at any t. Cash reserves earn a constant rate, $r \leq \rho$. Whenever $r < \rho$, keeping cash entails an opportunity cost.[5] In contrast with extant cash holdings models—in which the strict inequality $r < \rho$ is needed to depart from the corner solution featuring firms piling infinite cash reserves—I allow for the $r = \rho$ case. The cash reserves process satisfies:[6]

$$dC_t = \; rC_t dt + \mu_i dt + \sigma dZ_t - dD_t + f_t dN_t. \tag{2}$$

$dD_t \geq 0$ represents the instantaneous flow of payouts at time t. $f_t \geq 0$ denotes the instantaneous inflow of funds when financing opportunities arise, in which case management stores

[4]Section SA.1.3 of the Supplementary Appendix analyzes the corner case $\lambda \to \infty$, in which the firm faces no financing frictions as it immediately finds external financing upon searching. Absent financing frictions, financial policies become trivial. Thus, the model with financing frictions proves to be a more comprehensive analysis of the impact of stock illiquidity on firm value, which is particularly relevant in light of the empirical observation that firms with illiquid stocks are typically financially constrained, as discussed in the introduction. In the main body of the paper, I then focus on the case $\lambda < \infty$.

[5]This cost can be interpreted as a free cash flow problem (Jensen, 1986) or as tax disadvantages (Graham, 2000).

[6]Upon investing (i.e., the cash flow drift increases from μ to μ_+), the cost I is financed either with cash or external financing. Because the paper focuses on the decision of whether or not to invest (rather than on the investment timing), I do not explicitly spell out the outflow I in the dynamics of cash reserves.

the proceeds in the cash reserves. This assumption is consistent with the strong, positive correlation between equity issues and cash accumulation documented by McLean (2011) or Eisfeldt and Muir (2016). Notably, D and f are endogenous. Equation (2) implies that the firm's cash reserves increase with external financing, retained earnings, and the interest earned on cash, whereas they decrease with payouts and operating losses.

As in previous cash management models (see, e.g., HMM, BCW, DMRV), the cash reserves of the firm need to always remain nonnegative as an operating constraint. Subject to this constraint, management can distribute cash and liquidate the firm's assets at any time. Yet, liquidation is inefficient, as the recovery value of assets, denoted by ℓ, is smaller than the firm's first best, μ_i/ρ, due to liquidation costs. These costs erode a fraction, $1-\phi \in (0,1]$, of the firm's first best, so the liquidation value is $\ell = \phi\mu_i/\rho$. I denote by τ the endogenous time of liquidation.

Transacting the Firm Stocks The key departure from previous dynamic corporate finance models with financing frictions is the explicit consideration of stock transactions and the costs thereof. There are two types of traders: investors (who may buy, hold, and eventually sell the stock) and trading firms (or liquidity providers, which ease investors' trading).

Investors are ex-ante identical and infinitely lived. Each of them has measure zero and cannot short sell. Investors can be hit by liquidity shocks. As in previous contributions (e.g., Duffie et al., 2005, among others), liquidity shocks trigger a sudden need for liquidity that reduces the subjective valuation of the asset by a fraction χ. Thus, χ can be interpreted as a holding cost, i.e., as the opportunity cost of being locked into an undesired asset position because of take-it-or-leave-it investments, unpredictable financing needs, or unpredictable changes in hedging needs, for example. The liquidity shock vanishes once the shocked investor either sells his stock or bears the loss χ. Liquidity shocks are idiosyncratic, independent across investors, and occur at the jump times of a Poisson process $(M_t)_{t\geq 0}$ with intensity $\delta > 0$. In turn, non-liquidity-shocked shareholders have no immediate need to trade and, thus, are indifferent between keeping the stock or selling it at its fundamental value.[7]

[7]Following previous contributions (see, e.g., He and Milbradt (2014)), I assume that the mass of non-

Trading firms are agents who provide liquidity in the market for the stock—then helping liquidity-shocked shareholders unload their holdings. Throughout the paper, trading firms and liquidity providers will be used interchangeably. Trading firms are competitive, have no intrinsic demand for the firm's stock, and should be interpreted as pass-through intermediaries.[8] As I focus on smaller, less liquid stocks, I capture the realistic feature that trading firms may be distracted from the market of the firm's stock by trading opportunities arising in other markets. Indeed, the liquidity providers' engagement in several markets often goes to the detriment of smaller stocks—for instance, even NYSE specialists, who are obligated to provide liquidity in a well-defined set of securities, are at times inattentive in the market of smaller and less-traded stock, as documented by Corwin and Coughenour (2008). Notably, the potential inattention and, thus, the scant liquidity provision in the market of smaller stocks is indeed what motivates the idea of introducing designated market makers (DMM) paid directly by the firm issuing the security, as analyzed in Section 4.4.

If trading firms are actively engaged in the market of the stock, they spot trading opportunities in such market as soon as they arise. Trading firms are active on both sides of transactions. On the ask side, trading firms transact with non-liquidity-shocked investors, who are indifferent between staying out of the market or buying the stock at its fundamental value (i.e., they do not have an immediate need to trade). As a result, the gain to trading firms on this side of the transaction is null. On the bid side, trading firms transact with liquidity-shocked shareholders. Because shocked shareholders value the asset at a discount χ, trading firms can extract surplus from this side of transactions. The (gross) gain from transacting with shocked shareholders is denoted by η—i.e., η represents the bid-ask spread and is endogenously determined. In the spirit of Stoll (2000), trading firms bear a proportional order-flow cost ν on each round-trip transaction.

Alternatively, trading firms may pursue trading opportunities in other markets. Such opportunities are assumed to arise at the jump times of a Poisson process $(O_t)_{t\geq 0}$ with inten-

liquidity-shocked investors is larger than that of liquidity-shocked shareholders.

[8] For simplicity, the size of the trading firm sector is normalized to one. Similar effects (and results) could be obtained when endogenizing the mass of liquidity providers while assuming a given bid-ask spread. The corresponding analysis is available upon request.

sity ζ. When an opportunity arises, the associated payoff is an independent and identically distributed random variable W with probability density function g with positive support and mean $\omega > 0$.

The firm's problem with endogenous bid-ask spread Firm management maximizes equity value. Namely, cash retention and payouts (D), financing (f), liquidation (τ), and investment (I) are set to maximize:

$$V(c;\eta) = \sup_{(D,f,\tau,I)} \mathbb{E}\left[\int_0^\tau e^{-\rho t}\left(dD_t - f_t\, dN_t - \Phi(\eta_t,\chi)\, dM_t\right) + e^{-\rho\tau}\ell\right]. \tag{3}$$

As in previous cash management models (DMRV, BCW, or HMM), the first term in this equation is the discounted value of net payouts to shareholders, whereas the second term is the discounted value in liquidation. Differently from these works, net payouts to shareholders are not just the difference between the expected present value of all future dividends (dD_t) and the expected present value of all future gross issuance proceeds ($f_t\; dN_t$, which is akin to a negative payout), because frictions incurred in trading the firm stocks further drain the flow of net payouts to shareholders (represented by the term $\Phi(\eta_t,\chi)dM_t$). Namely, liquidity shocks—whose arrival rate is denoted by the Poisson process dM_t—lead to the loss $\Phi(\eta_t,\chi)$, which depends on the holding cost upon keeping the stock (χ) and the bid-ask spread incurred upon selling it (η_t).[9] In turn, because liquidity providers are competitive, the equilibrium bid-ask spread η_t leaves trading firms indifferent between providing liquidity in the market of the stock or pursuing trading opportunities in other markets. As the model solution illustrates, the endogenous bid-ask spread not only affects (as clear from equation (3)) but also reflects firm policies and value.

2.1 Discussion of the assumptions

The model nests trading frictions faced by the firm's shareholders into a dynamic corporate finance model with financing frictions. That is, the paper contributes to previous dynamic

[9]Because the loss $\Phi(\eta_t,\chi)$ is endogenous, it is then derived in the model solution, see section 3.

corporate finance models in this strand by studying the impact of stock illiquidity. At the same time, the paper contributes to models of the effects of liquidity demand/supply on asset valuations by explicitly focusing on the policies of the issuing firm. Whereas these models usually take the flow of dividends associated with a given stock as exogenous, the current paper endogenizes it. To keep the analysis simple, two assumptions are made. First, the trading costs borne by shocked (selling) investors are positive, whereas the costs borne by (buying) non-shocked investors are zero. This assumption is consistent with Brennan et al. (2012), who show that sell-order frictions are priced more strongly than buy-order ones.[10] Second, the model abstracts from asymmetric information about firm value. Indeed, recent evidence shows the importance of the non-information component of trading costs on asset prices (e.g., Chung and Huh, 2016).

The model recognizes that liquidity providers may be distracted from the market of the stock by trading opportunities in other markets. Whereas the modeling of these opportunities is stylized, it captures the idea that trading firms may weigh their decision to provide liquidity in the market of smaller stocks against other opportunities (whose value is independent of the value of the firm's stock). In fact, when liquidity providers span many markets, they are not active in all markets at the same time. Chakrabarty and Moulton (2012) report that market makers face attention constraints when trading opportunities or news arise in one markets, potentially reducing their liquidity provision in other markets.[11] As reported empirically, the liquidity providers' engagement in multiple stocks typically goes to the detriment of smaller stocks, see e.g., Corwin and Coughenour (2008).[12] For some small-cap stocks (or, even more so, micro-cap stocks), liquidity provision can be so scarce that their markets may exhibit an OTC structure, in which case the investors have to search for dealers and, upon finding them,

[10] Brennan et al. (2012) show that the pricing of illiquidity emanates principally from the sell side. The underlying idea is that agents seldom face needs to buy stock urgently, but unexpected needs for cash may force them to suddenly sell stocks.

[11] Schmidt (2019) documents a similar pattern for a broader set of institutional investors, whereby institutional investors are significantly less likely to trade in a given stock when there are many earnings announcements for other stocks on their watchlist.

[12] Furthermore, Coughenour and Harris (2005) find that large stocks represent about 76% of specialists' transactions and roughly 82% of combined specialists' revenue, in spite of the fact that large stocks are fewer compared to the universe of small stocks.

need to bargain over the terms of the trade.[13] I develop this extension in Section SA.2.1 of the Supplementary Appendix, in which the main model predictions continue to hold (i.e., a two-way relation between stock liquidity and firm policies and value continues to arise). To overcome the scarce liquidity provision in the market of smaller stocks, some exchanges have contemplated the idea of introducing designated market makers financed by the firm issuing the security as a way to improve their liquidity—a model application that I analyze in Section 4.4.

Given the paper's focus on firm policies (especially financial ones), it is crucial to assess that the results are not driven by some specific assumptions. The paper assumes that the firm faces uncertainty in its ability to raise fresh funds as HMM, an issue that is especially severe for small firms, as also pointed out by the U.S. SEC Advisory Committee in Small and Emerging Companies. As illustrated by the survey evidence in Lins et al. (2010), financing uncertainty is one of the top reasons behind corporate cash stockpiling. Thus, the model realistically allows firm management to accumulate earnings in cash reserves to hedge against this uncertainty and withstand operating losses. Yet, to assess the robustness to alternative modeling of the firm's financing frictions, I design two extensions. First, Section SA.1.1 of the Supplementary Appendix allows the firm to tap credit line availability. In fact, whereas small (or micro) firms find it too costly (or unfeasible) to tap bond financing, they usually access debt by borrowing from banks. The model predictions are shown to be robust to this extension. Second, Section SA.1.2 of the Supplementary Appendix assumes that the firm faces issuance costs whenever raising new equity, as in DMRV and BCW, again confirming the robustness of the main model predictions.

[13] Foucault et al. (2013) report that some exchanges contemplate different trading mechanism for different securities or for different sets of investors. For instance, they report that the London Stock Exchange contemplates different trading mechanisms according to a stock's trading volume and market capitalization—for less liquid stocks, it is a dealer market.

3 Model solution

Financing frictions, the bid-ask spread, and the dynamics of firm value As in previous cash management models, the benefit of cash decreases with cash reserves. Its (opportunity) cost is the wedge between the return required by the investors and the return on cash. Thus, I conjecture that there is a target cash level, C_V, at which the cost and benefit of cash are equalized. Above C_V, it is optimal to pay excess cash out to shareholders. Below C_V, shareholders retain earnings in cash reserves and search for financing.

Management can choose to liquidate when the firm holds positive cash reserves. However, because the firm is profitable in expectation and liquidation is costly, it is optimal to delay the liquidation time as much as possible.[14] In other words, subject to the operating constraint that cash reserves must be nonnegative, the firm is liquidated the first time that the cash reserves process hits $c = 0$, in which case it cannot cover its losses. Thus, the endogenous time of liquidation is:

$$\tau = \inf \{t \geq 0 : C_t \leq 0\} . \tag{4}$$

Assume first that the firm does not have any growth option.[15] Using standard arguments, firm value satisfies the following HJB equation:

$$\begin{aligned} \rho V(c;\eta) = (rc+\mu)\, V'(c;\eta) + \frac{\sigma^2}{2} V''(c;\eta) + \lambda \sup_f [V(c+f;\eta) - V(c;\eta) - f] \\ + \delta \Big[(1 - \min[\eta;\chi]) V(c,\eta) - V(c,\eta) \Big] . \end{aligned} \tag{5}$$

The left-hand side is the return required by the investors. The first two terms on the right-hand side represent the effect of cash retention and cash flow volatility on equity value. The third term represents the surplus from raising external financing, weighted by its likelihood. In Appendix A, the enterprise value $V(c) - c$ is shown to increase with c, so it is optimal to

[14] In fact, the drift in equation (1) is positive (meaning that the firm is viable in expectation), and cash flow shocks are transitory (i.e., the process in equation (1) follows an arithmetic Brownian motion). Thus, such shocks do not jeopardize the long-term prospects of the firm. Moreover, liquidation is costly as only a fraction of the present value of future cash flows is recovered, as per the definition of ℓ.

[15] Following Décamps and Villeneuve (2007) and HMM, solving for firm value when there are no growth option is auxiliary to studying the optimal investment rule, which is studied at the end of this section.

raise the cash buffer up to C_V whenever financing opportunities arise.[16] Thus, the optimal refinancing amount is $f(c, \eta) = C_V - c$. As shown in the following, C_V depends on the bid-ask spread and, thus, the optimal refinancing amount depends on the bid-ask spread too.

The novelty of equation (5) compared to previous cash management models is the last term on the right-hand side, which reflects the impact of liquidity shocks borne by the firm shareholders on firm value. Liquidity shocks are independent across investors, so a measure δdt of shareholders is shocked on each time interval. Shocked investors bear a loss that is the minimum between $\eta V(c)$ and $\chi V(c)$—i.e., if $\eta > \chi$, shocked shareholders are better off bearing the holding cost than trading. That is, the loss borne by shocked shareholders is $\Phi(\eta, \chi) = \min[\eta; \chi] V(c, \eta)$, where η is endogenous and derived next.

Deriving the bid-ask spread When providing liquidity in the market for the stock, trading firms are both on the bid and ask sides. Trading firms buy stocks from liquidity-shocked investors—from which they extract a proportional rent denoted by η—and sell stocks to non-liquidity-shocked investors—so they extract no rents from this side of transactions, as non-liquidity-shocked investors are indifferent between staying out of the market or buying the stock at its fundamental value. Thus, the expected net gain from liquidity provision in the market of the stock is given by:

$$\text{Expected net gain from providing liquidity} = \delta \left[-\left(1 - \eta\right) V(c; \eta) + V(c; \eta)(1 - \nu) \right]. \quad (6)$$

Transactions happen at rate δ at which shareholders are liquidity-shocked, and the ensuing payoff is reported in the square brackets of equation (6). Namely, the first term in the square brackets represents the price at which trading firms buy the stock from liquidity-shocked shareholders. The second term represents the price at which trading firms sell this claim to non-liquidity-shocked shareholders, net of the order-processing cost. Notably, the bid-ask spread η enters this equation directly—by affecting the price at which trading firms buy from

[16] The marginal value of cash satisfies $V'(c) \geq 1$, see Appendix A for a proof. This implies that the first derivative of $V(c) - c$ is non-negative. Clearly, it is not optimal to raise external financing to replenish the cash buffer beyond C_V, otherwise the excess cash would be paid out as dividend.

shocked shareholders—as well as indirectly—by affecting firm value.

Yet, trading firms may pursue trading opportunities in other markets. When so doing, the associated expected gain is given by:

$$\text{Expected net profit from opportunities in other markets} = \zeta \underbrace{\int_0^\infty W g(W) dW}_{\equiv \omega} \tag{7}$$

which is the probability-weighted payoff coming from opportunities arising in other markets. The term in the integral is the mean of the random variable W, denoted by ω. Competitive forces imply that the bid-ask spread is pinned down by equating the expected gain from liquidity provision to the expected gain from opportunities arising in other markets. I.e., equating (6) and (7) gives:

$$\eta(c; V) = \nu + \zeta\omega(\delta V(c; \eta))^{-1}. \tag{8}$$

This expression illustrates that the bid-ask spread needs to compensate liquidity providers for the order-processing cost and for forgone opportunities in other markets. As equation (8) illustrates, η depends on firm value. Yet, as equation (5) illustrates, firm value depends on η. That is, there is a two-way relation between V and η. In the following, to ease the notation, I simply use $\eta(c)$ and $V(c)$.

Endogenous liquidity and firm value As in previous cash management models, it is possible to show that firm value is increasing and concave in c in the presence of financing frictions—i.e., firm value increases with cash reserves, and the marginal value of cash is greater when the cash reserves are smaller (see Appendix A for a proof). In turn, monotonicity of firm value in the cash reserves leads to the following proposition.

Proposition 1 *There is at most one threshold* $\underline{C} \in [0, C_V]$ *such that the proportional loss borne by liquidity-shocked investors is equal to* $\eta(c)$ *for any* $c \in [\underline{C}, C_V]$ *whereas is equal to* χ *for any* $c \in [0, \underline{C}]$*.*

Proposition 1 shows that liquidity-shocked shareholders either never bear the holding cost χ (i.e., the threshold $\underline{C}$ does not exist) or they do when the firm's cash reserves are below the endogenous threshold $\underline{C}$. That is, if such threshold exists in $(0, C_V]$, liquidity-shocked shareholders bear the loss $\chi\delta V(c)$ if $c \in [0, \underline{C})$, whereas they bear the loss $\eta(c)\delta V(c)$ for any $c \in [\underline{C}, C_V]$.

Firm value is subject to the following boundary conditions. First, as explained above, the firm is liquidated when cash is exhausted and the firm cannot raise external funds. Thus,

$$V(0) = \ell \tag{9}$$

holds. Moreover, it is optimal to distribute all the cash exceeding C_V as payouts. Firm value is thus linear for any $c \geq C_V$: $V(c) = V(C_V) + c - C_V$. Subtracting $V(c)$ from both sides of this equation, dividing by $c - C_V$, and taking the limit $c \to C_V$ gives

$$\lim_{c \uparrow C_V} V'(c) = 1. \tag{10}$$

That is, it is optimal to start paying out cash when the marginal value of one dollar inside the firm equals the value of a dollar paid out to shareholders. The target cash level that maximizes shareholder value is determined by the super-contact condition,

$$\lim_{c \uparrow C_V} V''(c) = 0. \tag{11}$$

If $\underline{C} \in [0, C_V]$ exists, additional boundaries need to be imposed to warrant that firm value is continuous and smooth (see Appendix A for analytical details).

Finally, consider the firm's decision to invest in the growth opportunity. I denote by $V_+(c - I)$ the value of the firm after investment, in which case the firm enjoys a higher cash flow drift μ_+ after spending the cost I. Following Décamps and Villeneuve (2007) and HMM, I derive the zero-NPV investment cost I_V—i.e., the maximum amount that the firm is willing to spend to invest in the growth option. As proved in the Appendix A.1, the zero-NPV investment cost satisfies:

$$I_V = \frac{\mu_+ - \mu}{\rho + \delta\nu} - \left(1 - \frac{r}{\rho + \delta\nu}\right)(C_{V+} - C_V) \tag{12}$$

where C_{V+} denotes the target cash level after the growth option exercise. This expression illustrates that stock illiquidity affects the firm's decision to invest—namely, it erodes the amount shareholders are willing to pay to exercise the growth option. In the next section, I systematically analyze the effects of stock illiquidity, both analytically and quantitatively.

4 Model analysis

4.1 A special case: Exogenous bid-ask spread

To disentangle the strengths at play in the model, I start by investigating the special case in which the bid-ask spread is exogenous—i.e., the quantity η in equation (5) is given and constant. If $\eta < \chi$, equation (5) boils down to:

$$(\rho + \delta\eta)V(c) = (rc + \mu)\,V'(c) + \frac{\sigma^2}{2}V''(c) + \lambda[V(C_V) - V(c) - C_V + c]. \tag{13}$$

In line with the seminal work of Amihud and Mendelson (1986), the bid-ask spread leads shareholders to require a higher compensation to invest in the firm, as the left-hand side of this equation is increased by $\delta\eta$. This additional compensation is greater if the bid-ask spread is larger (i.e., higher η) or if shareholders need to trade more often (greater δ).

To analyze the effects of stock illiquidity, I define the following quantities. First, the firm's payout probability satisfies:

$$P^p(c, C_V) = \; E_c\left[e^{-\lambda\tau_d(C_V)}\right], \tag{14}$$

where $\tau_d(C_V)$ represents the first time that the cash reserves process, initially at a given $c < C_V$, reaches the target level C_V. Furthermore, the probability of liquidation while the firm is searching for external funds is given by:

$$P^l(c, C_V) = \ E_c\left[e^{-\lambda\tau(C_V)}\right] \tag{15}$$

and, complementarily, the probability of external financing is $P^f(c, C_V) = \ E_c\left[1 - e^{-\lambda\tau(C_V)}\right]$, where $\tau(C_V)$ represents the first time that the cash process, reflected at C_V, is absorbed at zero, as defined in equation (4). The next proposition summarizes the impact of an exogenous bid-ask spread on firm's decisions and outcomes (see Appendix B for proofs).

Proposition 2 *A positive bid-ask spread leads to:*

(1) A decrease in the target cash level—the greater the bid-ask spread η*, the smaller* C_V*;*

(2) An increase in the firm's payout probability, i.e., $P^p(c, C_V)$ *increases with* η*;*

(3) An increase in the firm's probability of liquidation and a decrease in the probability of external financing, i.e. $P^l(c, C_V)$ *increases with* η *and* $P^f(c, C_V)$ *decreases with* η*;*

(4) A decrease in the maximum amount that the firm is willing to pay to exercise the growth option, as the zero-NPV cost is (with C_{V+} *denoting the post-investment target cash level):*

$$I_V = \frac{\mu_+ - \mu}{\rho + \delta\eta} - (C_{V+} - C_V)\left[1 - \frac{r}{\rho + \delta\eta}\right]; \tag{16}$$

(5) A decrease in firm value, i.e., $V(c)$ *decreases with* η*.*

Claim (1) of Proposition 2 explains how the bid-ask spread affects corporate cash hoarding. The benefit of cash stems from providing financial flexibility to the firm facing financing frictions. The target cash level balances this benefit against the opportunity cost of cash. The analysis shows that the bid-ask spread affects both the benefit and the cost of cash. Indeed, as illustrated by equation (13), the bid-ask spread: (1) increases the cost of equity, then affecting the benefit of cash, and (2) increases the cost of cash, as it expands the wedge between the return required by the investors and the return on cash.[17] The proposition

[17]This model then delivers finite target cash levels even when r and ρ coincide. In previous dynamic cash management models, differently, holding cash is not costly if $r = \rho$ and, thus, a financially constrained firm would pile infinite cash reserves in such case.

shows analytically that the net effect of these strengths leads to a smaller target cash level compared to a benchmark case with no illiquidity. I.e., the bid-ask spread negatively affects the firm's ability to hold cash, consistent with the evidence in Nyborg and Wang (2021).

Cash retention and payout decisions are closely related. As illustrated by equation (14), the target cash level C_V affects the probability with which the firm pays out dividends. Hence, by affecting C_V, the bid-ask spread also affects the probability of payout. Specifically, claim (2) of Proposition 2 suggests that a firm pays out more dividends if its stock is traded at a larger bid-ask spread, as the target cash level is hit more often. In so doing, the firm compensates shareholders for the frictions borne when trading the stock. This finding is in line with Banerjee et al. (2007), who suggest that investors view stock market liquidity and dividends as substitutes. When a firm's bid-ask spread is small, investors can create dividends to themselves by cashing out their investment. When the bid-ask spread is large, investors require the firm to pay out more dividends.

Because the bid-ask spread increases the cost of internal and external equity, the firm's financial resilience is also affected. As illustrated by equation (15), the choice of the target cash level affects the time $\tau(C_V)$ at which the firm is liquidated. Indeed, Claim (3) of Proposition 2 shows that the greater a firm's bid-ask spread, the higher the firm's probability of liquidation and the lower the firm's probability of external financing. Proposition 2 then suggests that bid-ask spreads exacerbate firms' financial constraints and increase a firm's threat of forced liquidations—a finding consistent with Brogaard et al. (2017).

The bid-ask spread affects not only financial policies but also investment. Claim (4) of Proposition 2 suggests that a positive bid-ask spread leads to a decrease in the investment reservation price—that is, it reduces the maximum amount that the firm is willing to pay to exercise the growth option. Consider the investment reservation price when the bid-ask spread is zero and denote it by I^*.[18] If the investment cost lies in $[I_V, I^*]$, the growth option has negative NPV if the bid-ask spread is positive ($\eta > 0$), whereas it has positive NPV if the

[18] In this case, the zero-NPV cost would be $I^* = \frac{\mu_+ - \mu}{\rho} - (C_+^* - C^*)\left(1 - \frac{r}{\rho}\right)$, with C^* denoting the pre-investment target cash level when the bid-ask spread is zero ($\eta = 0$) and C_+^* denoting the post-investment target cash level.

bid-ask spread is zero. Thus, the positive bid-ask spread leads to underinvestment, a result that is empirically consistent with Campello et al. (2014) and Amihud and Levi (2022).

To summarize, a positive bid-ask spread adversely impacts the financial and investment policies of the firm. Claim (5) of Proposition 2 then concludes that it also decreases firm value, consistent with the evidence by Fang et al. (2009). As I show next, such a drop in firm value has important implications when the bid-ask spread is endogenous.

4.2 Endogenous bid-ask

As shown by Proposition 2 for the case with exogenous bid-ask spread, trading losses lead shareholders to require an additional compensation to invest in the firm. Such additional compensation constrains corporate policies—e.g., it makes it more costly to the firm to hold cash, which tightens the firm's financial constraints and increases the firm's probability of liquidation—and leads to a decrease in firm value. When allowing the bid-ask spread to be endogenous, such a drop in firm value is internalized by liquidity providers. Specifically, it leads to an increase in the equilibrium bid-ask spread that makes the liquidity providers' indifference condition binding. The greater bid-ask spread feeds back into firm value, then making the detrimental effects illustrated in Section 4.1 stronger. This two-way relation between the firm and the liquidity of its stock gives rise to a rich set of novel implications, which I analyze in the following.

Baseline parameterization Before turning to the model implications (which are gauged quantitatively too), I describe the baseline parameterization reported in Table 1. The risk-free rate ρ is set to 2%, and the return on cash is set to 1%. The resulting opportunity cost of cash is equal to 1%, as in BCW and DMRV. Because small firms tend to have lower sales and cash flows in the cross section (see, e.g., Fama and French, 2008), the drift $\mu = 0.05$ is set to be lower than the value used by DMRV and consistent with the bottom range of values in Whited and Wu (2006). Upon exercising the growth option, the drift is assumed to be 20% bigger (i.e., $\mu_+ = 0.06$). I set $\sigma = 0.12$, which is consistent with Graham et al. (2015)

and is higher than the value set by DMRV, as small firms have more volatile cash flows. I base liquidation costs on the estimates of Glover (2016) and set $\phi = 0.55$. The parameter λ is set to 0.75, which is consistent with the frequency of equity issues by small firms reported by Fama and French (2005). The intensity of the liquidity shock is set to $\delta = 0.7$, as in He and Milbradt (2014). The parameters related to trading firms' liquidity provision are varied extensively and chosen to convey realistic magnitudes of the bid-ask spreads for a small stock—under the baseline parameterization, the bid-ask spread ranges between 53 and 76 basis points.[19]

Liquidity provision affects corporate outcomes As shown in Section 4.1, the exogenous bid-ask spread affects corporate policies and outcomes. When the bid-ask spread is endogenous, variables that affect liquidity provision in the market of the stock—which are direct determinants of the bid-ask spread—impact corporate policies too, as formally shown by the next proposition.

Proposition 3 *If liquidity providers face a larger order-processing cost ν or a larger expected payoff from outside opportunities ω, the bid-ask spread rises and leads to: (1) a lower target cash level; (2) a higher payout probability; (3) a higher probability of liquidation and a lower probability of external financing; and (4) a decrease in firm value.*

Proposition 3 indicates that the costs of liquidity provision (being related to order processing or foregone opportunities) are passed on to the firm's shareholders and, through this channel, impact corporate policies and outcomes. Namely, if ν or ω increase, the bid-ask spread rises and, in turn, affects both the benefit and the cost of cash (as it affects both the cost of external and internal equity). This proposition shows that, on net, a greater ν or a

[19] Chung and Zhang (2014) report the median bid-ask spread for firms sorted by quintiles of market capitalization over the period 1993-2009 (calculated using TAQ data). They report that the median bid-ask spread of smaller quintile firms is 0.0195 for NYSE/AMEX stocks and 0.0501 for NASDAQ stocks. Yet, they also note that the bid-ask spread has decreased over time (see also Hasbrouck, 2009): The median bid-ask spread for (all capitalization) NYSE/AMEX stocks went from 0.0094 in 1993 to 0.0034 in 2009, and from 0.0346 in 1993 to 0.0067 in 2009 for NASDAQ stocks. In the model parameterization, I take a conservative approach and take a relatively low value for the bid-ask spread. In so doing, I show that even small bid-ask spreads can bear substantial impact on corporate policies and value.

greater ω lead the firm to keep less cash and pay out more to shareholders. Moreover, they also lead to an increase in the probability of forced liquidation and a decrease in the probability of external financing. Overall, a greater ν or ω decrease firm value. Notably, these effects are similar to those described in Proposition 2—i.e., the primitive variables affecting the bid-ask spread themselves impact corporate policies.

Table 2 quantifies these predictions by gauging how ν and ω affect firm choices and outcomes compared to the case in which the stock is perfectly liquid (i.e., the bid-ask spread is zero). When the order-processing cost is equal, for instance, to 0.2%—in which case the bid-ask spread is equal to 53 basis points for $c = C_V$—the target cash level decreases by about 10% compared to the case with perfect stock liquidity. Moreover, the probability of payout increases by 2.2% on average.[20] Overall, the firm is more financially constrained and faces a higher probability of liquidation—for $\nu = 0.2\%$, this probability increases on average by 1.4%.

Table 3 further investigates the impact of ν and ω on the firm's probability of liquidation at different levels of the cash reserves. It shows that an increase in the cost of liquidity provision raises the firm's probability of liquidation, and more so if the firm's cash reserves are smaller. Moreover, for a given cumulative shock, liquidation becomes relatively more likely if the bid-ask spread is larger. If trading the stock was costless (i.e., the bid-ask spread was zero, in which case the target cash level is denoted by C^*), a series of shocks reducing the cash buffer from $C^*/2$ to $C^*/4$ would increase the probability of liquidation from 1.97% to 14.11% (see the last line of Table 3). When instead the bid-ask spread is positive, a series of shocks reducing the cash buffer from $C_V/2$ to $C_V/4$ increase the probability of liquidation from 2.89% to 17.05% under our baseline parameterization. Moreover, the reduction from $C_V/2$ to $C_V/4$ would be caused by a cumulative loss that is 9.6% smaller. That is, firms traded at a larger bid-ask spread are less financially resilient.

Table 2 also shows that the costs incurred by liquidity providers have a substantial impact on the firm's investment decisions, consistent with the evidence in Goldberg (2020). Namely,

[20] To calculate the probabilities in this table, I follow HMM and calculate them for a cross-section of firms with cash reserves uniformly distributed between 0 and C_V.

they lead to a sharp reduction in the maximum price that the firm is willing to pay to increase the cash flow drift from μ to μ_+. Under the baseline parameterization, such a decrease in the maximum investment cost is about 7.6%. Overall, ν and ω have a substantial, detrimental effect on firm value, as quantified in the last column of Table 2—under the baseline parameterization, the decrease is about equal to 16.5%.

Identifying the indirect effects of shocks The intertwined relation between the firm and the liquidity of its stock implies that shocks directly affecting liquidity provision not only affect the bid-ask spread but also corporate policies and value. Similarly, shocks that arise within the firm affect the bid-ask spread too. Below I analyze these novel indirect effects.

Consider first the effect of shocks to primitive parameters affecting liquidity provisions, ν and ω. The model illustrates that such shocks affect the firm's bid-ask spread through a direct *and* an indirect channel. Indeed, an increase in ν or ω leads to a direct increase in the bid-ask spread that makes the liquidity providers' indifference condition binding, as clear from equation (8). This is the direct effect. Yet, as shown by Proposition (3), an increase in ν or ω also affects corporate policies and reduces firm value which, by equation (8), leads to a further increase in the equilibrium bid-ask spread. This is the indirect effect.

Table 4 analyzes the magnitude of such direct and the overall (direct plus indirect) effect when considering an increase in ν or ω with respect to the baseline parameterization. Under such parameterization, the bid-ask spread is equal to 53 basis points.[21] For example, consider an increase in ω by +0.004 compared to its baseline value reported in Table 1. Simply accounting for the direct effect described above (i.e., if firm value was independent of the bid-ask spread), such an increase in ω would increase the bid-ask spread by 22 basis points (i.e., to 75 basis points). When accounting for the indirect effect too—i.e., acknowledging that such shock also affects corporate policies and value—the bid-ask spread increases by 27 basis points (i.e., to 80 basis points). Similarly, when simply accounting for the direct effect, an increase in ν by +0.0025 compared to its baseline value would lead to an increase the bid-ask spread by 25 basis points (i.e., to 78 basis points). However, when accounting

[21] To fix ideas, I focus on the bid-ask spread at $c = C_V$. The bid-ask spread for any c is shown in Figure 1.

for the impact of ν on firm policies and value (i.e., the indirect effect), the bid-ask spread increases by 28 basis points (i.e., to 81 basis points). That is, the model shows that shocks making liquidity provision more costly lead to a sharper increase in the bid-ask spread when reflected into firm value and policies.

Consider next the impact of shocks originating within the firm, such as operating shocks. Negative (respectively, positive) operating shocks deplete (replenish) the cash reserves and decrease (increase) firm value. Concavity of firm value—which stems from the presence of financing frictions as in previous cash management models, as discussed in Section 3 (and proved in Appendix A)—implies that, for an operating shock of a given size, firm value is more sensitive if the shock is negative (than if it is positive). The next result follows.

Proposition 4 *For an operating shock of a given size, the resulting change in the bid-ask spread is greater if such shock is negative.*

Proposition 4 shows that stock illiquidity reacts asymmetrically to negative or positive cash flow shocks—i.e., the increase in the bid-ask spread following a negative shock is greater in absolute magnitude than the decrease following a positive shock of the same size. Figure 1 supports this pattern as the bid-ask spread is a steeper function of c as the firm gets closer to the liquidation boundary. Suppose, for example, that the firm holds half of its target cash level (i.e., $c = C_V/2$). A shock wiping out 25% of the target cash reserves would lead to a 4 basis point increase in the bid-ask spread, whereas a positive shock of the same magnitude would only lead to a 2 basis point decrease. Moreover, a shock wiping out 40% of the target cash level would lead to a 9 basis point increase in the bid-ask spread, whereas a positive shock of the same magnitude would lead to a 3 basis point decrease. Consistently, Hameed et al. (2010) show that liquidity responds asymmetrically to shocks to asset values, deteriorating more sharply after negative ones.

A natural question arises as for which firms the interplay between the firm and the liquidity of its stock is stronger. Table 5 points to smaller firms—in the model, exhibiting a lower μ.[22] Notably, the table illustrates three important points. First, for lower μ, the firm

[22]Empirically, firm size is typically gauged through firm sales, whose model counterpart is μ.

is not only less valuable, but also exhibits a larger bid-ask spread—this is consistent with the empirical observation that smaller firms are typically less liquid. Second, the relation between μ and the bid-ask spread is non-linear—namely, a further decline in μ when it is lower leads to a relatively larger increase in the bid-ask spread. Third, this translates into a sharper drop in firm value compared to the counterfactual environment with perfect liquidity (see the last column of the table). The reason is that the ensuing deterioration in illiquidity amplifies the impact of a decrease in μ on firm value. Lastly, the bottom panel of this table shows that such deterioration in liquidity and firm value is even bigger if ν or ω are larger.

Reassessing standard determinants of cash reserves and liquidation probability The analysis so far illustrates how shocks arising within the firm or in the market of the stock can bear indirect effects, which have been overlooked by the previous literature. Given the focus on cash reserves, I now investigate the impact of shocks to major determinants of corporate cash holdings like the firm's access to external financing and to cash flow volatility, see Opler et al. (1999) and Bates et al. (2009).

Consider first a tightening in the firm's access to external financing—in the model, this shock is captured by a decrease in the parameter λ.[23] As a direct effect, the firm's precautionary demand for cash increases—and so does the firm's target cash level—and firm value declines. Yet, when the bid-ask spread and firm value are jointly determined, such a direct effect simultaneously leads to an increase in the bid-ask spread that makes the liquidity providers' indifference condition binding. The increase in the bid-ask spread inflates the return required by the investors and the opportunity cost of cash, a strength that pushes the target cash level down. That is, this indirect effect hampers the firm's ability to accumulate cash exactly when its demand for cash increases (precisely because access to external financing is tighter)—in other words, it makes it costlier to the firm to keep cash when it needs it *the most*. Thus, when bid-ask spread and firm value and policies affect each other, a tightening in external financing leads the target cash level to increase by less than if stock

[23] Indeed, a decrease in λ implies that the firm's access to external financing is more uncertain (or, equivalently, it takes a longer time in expectation to secure financing upon searching).

liquidity was perfect. Consistently, Table 6 shows that the gap between C_V and C^* (i.e., the target cash level if the bid-ask spread is zero) widens as λ decreases. That is, while C^* increases as a direct effect of the tightening in capital supply, C_V increases by less.

Shocks to the firm's access to financing naturally impact its probability of liquidation. Indeed, if access to financing tightens (i.e., λ decreases), the firm's probability of liquidation increases as a direct effect. As just explained, such shock also increases the opportunity cost of cash as an indirect effect, leading the firm to keep a smaller cash reserves compared to the case with perfect liquidity. This, in turn, pushes the probability of liquidation higher, beyond the impact solely driven by the aforementioned direct effect. Consistently, Table 6 shows that the probability of liquidation between the case with endogenous bid-ask spread and the case with perfect stock liquidity widens as λ decreases. That is, the interplay between stock liquidity and firm value amplifies the impact of a tightening in capital supply on the firm's probability of liquidation.

Similar effects arise in the wake of an increases in the volatility of the firm's cash flows, σ. As a direct effect, an increase in σ expands the firm's precautionary demand for cash and decreases firm value—a standard result in cash management models.[24] When the bid-ask spread and firm value are jointly determined, however, such a direct effect simultaneously leads to an increase in the bid-ask spread that makes the liquidity providers' indifference condition binding. The cost of cash then increases, which gives rise to a decline in the firm's target cash level that partially offsets the direct increase. In this case too, this indirect effect hampers the firm's ability to accumulate cash when it is more needed (in this case, because cash flows are more volatile). As shown in the bottom panel of Table 6, the gap between C_V and C^* widens as σ rises: Whereas C^* increases as a direct effect of the increase in σ, C_V increases by less. Importantly, this effect amplifies the impact of an increase in σ on the firm's probability of liquidation. In fact, while an increase in σ directly inflates the probability of liquidation, the lower firm's reliance on cash reserves increases such probability further. Consistently, Table 6 shows that the gap between the probability of liquidation with

[24]Because firm value is concave in cash reserves due to the presence of financing frictions (as discussed in Section 3 and proved in Appendix A), an increase in σ decreases firm value.

endogenous liquidity versus the benchmark with perfect liquidity widens as σ increases. That is, shocks to cash flow volatility have an amplified impact on the firm's probability of liquidation in the presence of the two-way relation between the firm and its stock liquidity.

This analysis then illustrates that the interplay between the firm and the liquidity of its stock makes firms less financially resilient—indeed, such interplay amplifies the impact of adverse shocks on the probability of forced liquidations.

4.3 Testable predictions

This paper provides a theoretical framework that delivers a unified explanation for a set of empirical regularities relating stock market liquidity and corporate policies. As shown by Proposition 2 and 3, the paper shows that firms whose stocks are traded at a higher bid-ask spread hold less cash (Nyborg and Wang, 2021), pay out more dividends (Banerjee et al., 2007), have a greater default probability (Brogaard et al., 2017), invest less (Campello et al., 2014; Amihud and Levi, 2022), and overall are less valuable (Fang et al., 2009). On top of these results, the analysis suggests novel testable predictions that exploit the interplay between bid-ask spread and the policies and value of the issuing firm.

First, the model suggests that shocks exacerbating the liquidity of a firm's stock bear an amplified impact when they also affect firm policies and value. A test of this prediction could then exploit exogenous (unexpected) shocks to the costs borne by liquidity providers. As liquidity providers are typically active in several stocks, such shocks would affect a pool of stocks, which would then help exploit a cross-sectional dimension that could validate the model mechanism.[25] Indeed, the model suggests that the smaller stocks in the affected portfolio should be more exposed to the two-way relation between liquidity and firm value and, thus, should experience both the direct *and* the indirect effect described in Section 4.2 (see Table 4). Following the shock, such stocks should experience a sharper change in their corporate policies. Namely, they would keep less cash and pay out dividends more often.

[25] In the content of the real effects of financial markets, Bond et al. (2012) indeed point to exploiting cross-sectional heterogeneity to identify firms that might be insulated from the effect of illiquidity and any potential feedback effect.

Moreover, they would become less resilient to negative operating shocks, then exhibiting an increase in their liquidation probability. In addition, such firms would curtail their investment and become less valuable. Testing these effects would validate the mechanism at play in the model. Notably, the gap in the increase in the bid-ask spread between the firm exhibiting the largest deviations in corporate policies versus those exhibiting no changes would help gauge the extent of the amplification effect. In fact, the firms exhibiting no changes in corporate policies should be insulated by the two-way relation described in the paper.

Second, the model suggests that empirical tests aimed at examining the determinants of corporate cash holdings should be revisited to account for their simultaneous impact on stock liquidity. The analysis indeed indicates that standard determinants such as the firm's access to external financing and cash flow volatility affect cash reserves through a benefit *and* a cost channel—the latter being driven by their impact on the firm's bid-ask spread. As discussed, the latter effect is novel to the literature and, importantly, hampers the firm's ability to accumulate cash when the firm needs it *the most* (i.e., when the firm's demand for cash increases). Thus, empirical tests disentangling such benefit and cost channels by controlling for the simultaneous impact on stock liquidity would shed new light on the quantitative impact of such cash determinants.

Third, and related to the previous point, the model suggests that exogenous shocks to the firm's access to external financing or cash flow volatility have an amplified impact on the firm's probability of liquidation when firm choices and stock liquidity are jointly determined. As a result, the two-way relation between the firm and its bid-ask spread makes firms more fragile and more vulnerable to forced liquidations in the wake of such adverse shocks. Empirical work could then exploit exogenous shocks to the firm's access to external financing—which directly affect the firm's probability of liquidation—and investigate how illiquidity plays a role in amplifying such probability.[26] Again, the dimension of cross-sectional heterogeneity could help gauge the extent of the amplification, as it would help identify firms that are insulated from the effect of illiquidity (and, thus, its impact on the

[26]For instance, Duchin et al. (2010) and Campello et al. (2010) investigate the effects of shocks to the supply of financing using the 2007-2009 financial crisis as a laboratory.

probability of liquiditation).

Fourth, the analysis shows that, to gauge the severity of a firm's financial constraints, empiricists should account for stock illiquidity. Indeed, the analysis in the paper indicates that stock liquidity affects the firm's probability of financing and of liquidation, then affecting its degree of financial constraints. While measures of financial constraints typically harness firm characteristics as useful predictors of financial constraints (such as the WW index suggested by Whited and Wu (2006) or the SA index of Hadlock and Pierce (2010)), this paper suggests that the liquidity of the firm's stock could improve the predictive power of such measures.

4.4 Application: Firm-funded Designated Market Makers (DMM)

Competitive market forces may, at times, lead to inefficient liquidity provision, see e.g. Bessembinder et al. (2015) or Venkataraman and Waisburd (2007).[27] This is especially the case for small stocks, an issue that has raised the attention of policymakers. For example, the recommendations of the SEC Advisory Committee on Small and Emerging Companies are based on the idea that competitive market forces may break down when it comes to small (or micro) firms. Partially addressing this issue, stock markets in several European countries have contemplated a contract whereby listed firms pay a DMM to enhance the liquidity of their stock and maintain their bid-ask spread below a given (contractual) threshold.

I extend the model to study the desirability of this policy provision from the perspective of financially constrained firms. Consider the contract between a listed firm and a DMM. As described by Bessembinder et al. (2015) or Menkveld and Wang (2013), I assume that the DMM is required to keep the bid-ask spread within a specific width in exchange for a periodic rent that is paid by the firm. Consistently, I assume that the firm pays the DMM a flow payment Γ on any dt to improve the liquidity of the firm's stocks and keep the bid-ask below a given level $\bar{\eta}$, which is assumed to be smaller than χ (to consider the relevant case).

[27] Bessembinder et al. (2015) show that competitive liquidity provision in secondary markets is associated with reduced welfare and a discounted secondary market price that can potentially dissuade IPOs.

In this setup, the equilibrium bid-ask spread satisfies (see Appendix D):

$$\eta(c;V) = \min\left[\nu + (\zeta\omega - \Gamma)(\delta V(c))^{-1}\,,\ \bar{\eta}\right]. \tag{17}$$

This equation differs from equation (8) in two main aspects. First, the bid-ask spread cannot exceed the contractual maximum $\bar{\eta}$. Second, the rent paid by the firm to the DMM leads to a reduction in the bid-ask spread.

When the firm enters the DMM contract, firm value satisfies the following equation:

$$\begin{aligned}\rho V(c;\eta) = &(rc + \mu - \Gamma)\, V'(c;\eta) + \frac{\sigma^2}{2} V''(c;\eta) + \lambda \sup_f [V(c+f;\eta) - V(c;\eta) - f] \\ &+ \delta\,[(1-\eta)V(c,\eta) - V(c,\eta)]\,. \end{aligned} \tag{18}$$

Differently from the setup with no DMM (see equation (5)), the bid-ask spread is lower and capped at $\bar{\eta}$ under the DMM contract, meaning that a lower cost of illiquidity is passed on to shocked shareholders (through the last term of equation (18)). Yet, the first term on the right-hand side of equation (18) illustrates that the payment corresponded by the firm to the DMM, Γ, drains the firm's periodic cash flow. The following result is shown in Appendix D.

Proposition 5 *On net, entering the DMM contract does not increase the value of financially constrained firms if it entails a periodic fee paid by the firm to the DMM.*

Firms face a tradeoff when considering the DMM contract requiring a periodic fee. On the positive side, entering the contract reduces the bid-ask spread borne by firm shareholders, which positively affects the firm's financial policies and outcomes. On the negative side, entering the DMM contract drains the firm's cash flows due to the periodic fee corresponded to the DMM, which in turn makes the firm more financially constrained. Because the marginal value of cash is greater than one for financially constrained firms (as proved in Appendix A), cash is more valuable inside the firm than if used to fund the DMM. As a result, the negative effect dominates the positive effect.

Figure 2 compares the equilibrium bid-ask spread and firm value when the firm does and does not enter the DMM contract. The top panel shows that when the firm enters the contract, the equilibrium bid-ask spread decreases, which has a positive effect on firm value. Yet, the firm has to give up some of its revenues to fund the DMM. On net, the bottom panel of Figure 2 shows that firm value is almost unchanged, being slightly smaller if the firm enters the contract, consistent with Proposition 5.

While the analysis shows that paying a periodic flow to the DMM makes the contract suboptimal to constrained firms, Section SA.2.2 of the Supplementary Appendix explores how a one-off payment would affect this conclusion. Under such an alternative provision, there is a maximum amount that firms would be willing to bear at the outset to enter the DMM contract. Naturally, firms would need to accumulate a sufficient cash reserves before entering such contract, or they would have to wait for external financing opportunities. In other words, firms that are severely liquidity constrained would find it hard to enter such contract, and would need to wait for their liquidity position to improve. Overall, the design of DMM contracts is crucial to make it attractive to financially constrained firms.

5 Concluding remarks

This paper develops a model that sheds light on the two-way relation between a stock's bid-ask spread and the policies and value of the issuing firm. The model shows that bid-ask spreads increase the firms' cost of capital *and* the opportunity cost of cash. As such, they make firms more financially constrained, more exposed to forced liquidations, less prone to invest, and less valuable. The model shows that these outcomes get reinforced when internalized by liquidity providers, leading to a wider bid-ask spread and lower firm value. This mechanism implies that frictions faced by liquidity providers are passed on to the firm's investors and, through this channel, have an impact on the policies, values, and survival rates of small firms. Overall, this two-way relation implies that shocks arising within the firm or in the market for its stock have more nuanced impacts than previously understood. More generally, the model suggests that the architecture of secondary market transactions has a prime effect on corporate decisions, especially for firms that face severe financing frictions.

Appendices

A Proof of the Results in Section 3

Two separate cases are considered, as discussed below.

Case $\eta(c) < \chi$ for all $c \in [0, C_V]$. When this is the case, the expression for the optimal bid-ask spread (equation (8)) is substituted into equation (5) and gives:

$$(\rho + \delta\nu)V = (rc + \mu)\, V' + \frac{\sigma^2}{2}V'' + \lambda\left[V(C_V) - C_V + c - V(c)\right] - \zeta\omega. \tag{19}$$

for any $c \leq C_V$. Firm value is then solved subject to the boundary condition at the liquidation threshold and at C_V, as reported in the main text. It is possible to show that $V(c)$ is increasing and concave in c, as shown in the next Lemma.

Lemma 6 *$V'(c) > 1$ and $V''(c) < 0$ for any $c \in [0, C_V)$.*

Proof. Simply differentiating equation (19) one gets

$$(\rho + \lambda + \delta\nu - r)\, V'(c) = V''(c)\,(rc + \mu) + \frac{\sigma^2}{2}V'''(c) + \lambda.$$

By the conditions $V'(C_V) = 1$ and $V''(C_V) = 0$, it follows that $V'''(C_V) = \frac{2}{\sigma^2}\,(\rho + \delta\nu - r) > 0$ as $r < \rho$. Thus, there exists a left neighborhood of C_V such that for any $c \in (C_V - \epsilon, C_V)$, with $\epsilon > 0$, the inequalities $V'(c) > 1$ and $V''(c) < 0$ hold. Toward a contradiction, I assume that $V'(c) < 1$ for some $c \in [0, C_V - \epsilon]$. Then there exists a point $C_c \in [0, C_V - \epsilon]$ such that $V'(C_c) = 1$ and $V'(c) > 1$ over (C_c, C_V), so

$$V(C_V) - V(c) > C_V - c \tag{20}$$

for any $c \in (C_c, C_V)$. For any $c \in (C_c, C_V)$ it must be also that

$$V''(c) = \frac{2}{\sigma^2}\left\{(\rho + \lambda + \delta\nu)\, V(c) - [rc + \mu]V'(c) - \lambda(V(C_V) + c - C_V) + \zeta\omega\right\}$$

Using (20), jointly with $V(C_V) = \frac{rC_V + \mu - \zeta\omega}{\rho + \delta\nu}$, it follows that

$$V''(c) < \frac{2}{\sigma^2}\left\{(\rho + \delta\nu)(V(C_V) + c - C_V) - rc - \mu + \zeta\omega\right\} = \frac{2}{\sigma^2}(c - C_V)(\rho + \delta\nu - r) < 0.$$

This means that $V'(c)$ is decreasing for any $c \in (C_c, C_V)$, which contradicts $V'(C_c) = V'(C_V) = 1$. It follows that C_c cannot exists. So, $V'(c) > 1$ and $V''(c) < 0$ for any $c \in [0, C_V)$, and the claim follows. ■

Case $\eta(c) < \chi$ does not hold for some $c \in [0, C_V]$. In this case, there is a threshold $\underline{C}$ such that, for any $c \in [\underline{C}, C_V]$, $\eta(c) < \chi$ and firm value satisfies equation (19). Conversely, for $c \in [0, \underline{C}]$, shocked shareholders bear the holding cost χ, and firm value satisfies:

$$(\rho + \delta\chi)\, V = (rc + \mu)\, V' + \frac{\sigma^2}{2} V'' + \lambda \left[V(C_V) - C_V + c - V(c) \right]. \tag{21}$$

I now prove Proposition 1.

Proof of Proposition 1 By definition, $\eta(\underline{C}) = \chi$. Using equation (8), this equality holds if firm value is equal to $V(\underline{C}) = \frac{\zeta\omega}{\delta(\chi-\nu)} \equiv \underline{V}$. Towards a contradiction, suppose that there are two thresholds $\underline{C}_a < \underline{C}_b$ for which $\eta(\underline{C}_a) = \eta(\underline{C}_b) = \chi$ and, for any $c > \underline{C}_b$, $\eta(c) < \chi$. Thus, equation (21) holds over $[\underline{C}_a, \underline{C}_b]$, whereas equation (19) holds over $[\underline{C}_b, C_V]$. As shown in Lemma 6, $V' > 1$ and $V'' < 0$ for any $c > \underline{C}_b$. Yet, $V(\underline{C}_a) = V(\underline{C}_b) = \underline{V}$ would mean that $V'(c)$ goes below 1 (and below 0) for some $c \in [\underline{C}_a, \underline{C}_b]$. The strict concavity of $V(c)$ over $[\underline{C}_b, C_V]$ and the boundary condition $V'(C_V) = 1$ means that there should be a maximum $C_m \in [\underline{C}_a, \underline{C}_b]$ for V', such that $V'(C_m) > 1$, $V''(C_m) = 0$ and $V'''(C_m) < 0$. Differentiating equation (21) gives

$$V''(c)[rc + \mu] + V'''(c)\frac{\sigma^2}{2} - V'(c)(\rho + \delta\chi - r) + \lambda(1 - V'(c)) = 0.$$

Then, $V'''(C_m)\frac{\sigma^2}{2} = (\rho + \delta\chi - r)V'(C_m) + \lambda(V'(C_m) - 1) > 0$, which contradicts the existence of such a maximum C_m for $V'(c)$. It follows that there is not such a maximum for V'. Thus, there is at most one threshold $\underline{C}$. In particular, exploiting the expression $\underline{V}$, such a threshold exist if the inequality $\frac{\zeta\omega}{\delta(\chi-\nu)} \leq \ell$ holds. The claim follows. ■

Continuity and smoothness at $\underline{C}$ mean that the system of equations (19) and (21) is solved subject to the following conditions:

$$\lim_{c \uparrow \underline{C}} V(c) = \lim_{c \downarrow \underline{C}} V(c) \quad \text{and} \quad \lim_{c \uparrow \underline{C}} V'(c) = \lim_{c \downarrow \underline{C}} V'(c)$$

on top of the boundary conditions at the liquidation and payout threshold ($V(0) - \ell = \lim_{c \uparrow C_V} V'(c) - 1 = \lim_{c \uparrow C_V} V''(c) = 0$). In this case too, I can show that firm value is concave over any $c \in [0, C_V]$.

Lemma 7 *In the case in which shocked shareholders bear the opportunity cost χ for $c < \underline{C}$, with $\underline{C} \in [0, C_V]$, $V'(c) > 1$ and $V''(c) < 0$ for any $c \in [0, C_V)$ too.*

Proof. Using arguments similar to those in Proposition 6, it is possible to show that there exists a left neighborhood of C_V such that for any $c \in (C_V - \epsilon, C_V)$, with $\epsilon > 0$, the inequalities $V'(c) > 1$ and $V''(c) < 0$ hold. Toward a contradiction, I assume that $V'(c) < 1$ for some $c \in [0, C_V - \epsilon]$. Then, there should be a point $C_c \in [0, C_V - \epsilon]$ such that $V'(C_c) = 1$ and $V'(c) > 1$ over (C_c, C_V), so $V(C_V) - V(c) > C_V - c$ for any $c \in (C_c, C_V)$. The point C_c could belong either to the interval $[0, \underline{C}]$ or in the interval $[\underline{C}, C_V]$. In the first case, I can apply arguments similar to those used in Lemma 6. Consider the second case, featuring $0 < C_c < \underline{C}$. Should such point C_c exist, the strict concavity of $V(c)$ over $[\underline{C}, C_V]$ means that there should be a maximum $C_m \in [C_c, \underline{C}]$ for the first derivative over the interval $(C_c, \underline{C})$, such that $V'(C_m) > 1$, $V''(C_m) = 0$ and $V'''(C_m) < 0$. Using same arguments as in the proof to Proposition 1, such point cannot exist, and the claim follows. ■

A.1 Deriving the zero-NPV investment cost

I now derive the expression for the zero-NPV investment cost reported in equation (12). Exploiting the dynamic programming result in Décamps and Villeneuve (2007) and HMM, the growth option has a non-positive NPV if and only if $V(c) > V_+(c-I)$ for any $c \geq 0$, where $V_+(c-I)$ is the value of the firm after investment. To derive the zero-NPV cost, I rely on the following lemma.[28]

Lemma 8 *$V(c) \geq V_+(c-I)$ for any $c \geq I$ if and only if $I \geq I_V$, where I_V satisfies the expression reported in equation (12).*

Proof. I define $\bar{c} = \max[C_V, I + C_{V+}]$. The inequality $V(c) \geq V_+(c-I)$ for $c > \bar{c}$ means that $c - C_V + V(C_V) \geq c - C_{V+} - I + V_+(C_{V+})$. Using the definition of I_V, the former inequality is equivalent to the inequality $I \geq I_V$, by straightforward calculations.

To prove the sufficient condition, I can just prove that $V(c) \geq V_+(c - I_V)$ for any $c \geq I_V$. I exploit the inequalities $C_V < C_{V+} + I_V$ and $\mu_+ - \mu - rI_V > 0$ (these inequalities stem from a slight modification of Lemma C.3 in HMM, so I omit the details). For $c \geq C_V$, the following inequality

$$V_+(c - I_V) \leq V_+(C_{V+}) + c - I_V - C_{V+} = c - C_V + V(C_V) = V(c)$$

holds. The first inequality is due to the concavity of V_+. The first equality is given by the definition of I_V, whereas the second equality is due to the linearity of V above C_V. I now need to prove the result for $c \in [I_V, C_V]$. To this end, I define the auxiliary function $u(c) = V(c) - V_+(c - I_V)$. The function $u(c)$ is positive at C_V as argued above, $u'(C_V) < 0$ and $u''(C_V) > 0$. On the interval of interest it satisfies:

$$\begin{aligned}(\rho + \delta\nu + \lambda)u(c) =& (rc + \mu)u'(c) + \frac{\sigma^2}{2}u''(c) + (\mu + rI_V - \mu_+)V'_+(c - I_V) \\ &+ \lambda(V(C_V) - C_V - V_+(C_{V+}) + C_{V+} + I_V)\end{aligned}$$

where the last term on the right hand side is zero by the definition of I_V, while the third term is negative. Then, the function cannot have a positive local maximum here, because otherwise $u(c) > 0$, $u''(c) < 0 = u'(c)$, and the ODE above would not hold. Jointly with the fact that $u(C_V)$ is positive, decreasing, and convex means that the function is always decreasing on this interval. Then, $u(c)$ is also always positive, and the claim holds. ■

B Proof of the Results in Section 4.1

When the bid-ask spread is exogenous, equation (5) boils down to equation (13) in the main text. In this case too, it is optimal for the firm to raise funds up to the target cash level, as explained in Section 3.

B.1 Proof of Proposition 2

B.1.1 Claim (1): Monotonicity of the target cash threshold

I express the function $V(c)$ as a function of X, denoting the threshold satisfying $V'(X, X) - 1 = V''(X, X) = 0$. To prove the claim, I exploit the following auxiliary results.

[28] For simplicity, I focus on the case in which $\eta(c) < \chi$ for any $c \leq C_V$.

Lemma 9 *The function $V(c, X)$ is decreasing in X.*

Proof. To prove the claim, I take $X_1 < X_2$, and I define the auxiliary function $k(c) = V(c, X_1) - V(c, X_2)$, that satisfies

$$(\rho + \delta\eta + \lambda)k(c) = (rc + \mu)k'(c) + 0.5\sigma^2 k''(c) + \lambda(X_1 - X_2)[r/(\rho + \delta\eta) - 1] \tag{22}$$

for any $c \in [0, X_1]$. By calculations, the function is positive at X_2 as $k(X_2) = (X_1 - X_2)[r/(\rho+\delta\eta) - 1] > 0$. By the definition of X_1 and X_2, the function $k(c)$ is decreasing and convex for $c \in [X_1, X_2)$. Therefore, $k(X_1) > 0$. Consider now the first derivative of the previously defined function, $k'(c)$, which satisfies

$$(\rho + \delta\eta + \lambda - r)k'(c) = (rc + \mu)k''(c) + 0.5\sigma^2 k'''(c) \tag{23}$$

simply exploiting equation (22). Note that $k'(c)$ does not have a positive local maximum nor a negative local minimum, otherwise the equation (23) wouldn't hold (respectively, $k'(c) > 0 = k''(c) > k'''(c)$ and $k'(c) < 0 = k''(c) < k'''(c)$ at a positive maximum and at a negative minimum). As k is convex at X_1, this means that k' is increasing at X_1, and therefore it must be negative for any $c \in [0, X_1]$. Jointly with $k(X_1) > 0$, this means that $k(c) > 0$ for any $c \in [0, X_2]$. The claim follows. ■

Lemma 10 *For a given payout threshold X and two given $\eta_1 > \eta_2$, $V(c, X, \eta_2) > V(c, X; \eta_1)$ holds for any $c \in [0, X]$.*

Proof. I define the auxiliary function $h(c) = V(c, X; \eta_2) - V(c, X; \eta_1)$. I need to prove that, for a given payout threshold X, $h(c) > 0$ for any $c \in [0, X]$. At X, the function is positive as

$$h(X) = (rX + \mu)\left(\frac{1}{\rho + \delta\eta_2} - \frac{1}{\rho + \delta\eta_1}\right) = (rX + \mu)\frac{\delta\eta_1 - \delta\eta_2}{(\rho + \delta\eta_1)(\rho + \delta\eta_2)} > 0,$$

as $h'(X) = h''(X) = 0$. In addition, the function satisfies

$$[rc + \mu]h'(c) + \frac{\sigma^2}{2}h''(c) - (\rho + \lambda + \delta\eta_2)h(c) + \lambda h(X) = (\delta\eta_2 - \delta\eta_1)V(c, X; \chi_1)$$

and the right hand side is negative. Differentiating gives $[rc + \mu]h''(c) + \frac{\sigma^2}{2}h'''(c) - (\rho + \lambda + \delta\eta_2 - r)h'(c) = (\delta\eta_2 - \delta\eta_1)V'(c, X; \chi_1)$. At X, I get $\frac{\sigma^2}{2}h'''(X) = \delta\eta_2 - \delta\eta_1$, meaning that $h'''(X) < 0$. This means that the second derivative is decreasing in a neighbourhood of X, so one has $h''(c) > 0$ in a left neighbourhood of X (recall that $h''(X) = 0$). In turn, this means that $h'(c)$ is increasing in such a neighbourhood of X, then implying that $h'(c) < 0$ in a left neighbourhood of X. Now I need to prove that the function is decreasing for any c smaller that X. Note that, by the ODE above, $h'(c)$ cannot have a negative local minimum. As $h'(X) = 0$ and it is negative and increasing in a left neighbourhood of X, this means that $h'(c)$ should be negative for any $c < X$, so $h(c)$ is always decreasing. As it is positive at X, it means that it should be always positive, so $h(c) > h(X) > 0$ so it is positive for any $c < X$. ■

Exploiting the results above, I can prove the following lemma.

Lemma 11 *For any $\eta_1 > \eta_2$, $C_V(\eta_1) < C_V(\eta_2)$.*

Proof. The payout thresholds $C_V(\eta_1)$ and $C_V(\eta_2)$ are the unique solution to the boundary conditions $V(0, C_V(\eta_2); \eta_2) - \ell = 0 = V(0, C_V(\eta_1); \eta_1) - \ell$. Exploiting the result in Lemma 10, I now take, for instance, $X = C_V(\eta_1)$. It then follows that

$$V(0, C_V(\eta_1); \eta_2) - \ell \ > \ 0 \ = \ V(0, C_V(\eta_1); \eta_1) - \ell.$$

As V is decreasing in the payout threshold, this means that $C_V(\eta_1) < C_V(\eta_2)$ to get the equality $\ell - V(0, C_V(\eta_2); \eta_2) = 0$. The claim follows. ■

The next results stem from Lemma 11.

Corollary 12 *When the bid-ask spread is positive, the target cash level is lower than in the benchmark case with no bid-ask spread, i.e.* $C_V < C^*$.

Note also that all the results in this section can be extended for two parameters $\delta_1 > \delta_2$. The following result is then straightforward.

Corollary 13 *For any* $\delta_1 > \delta_2$, $C_V(\delta_1) < C_V(\delta_2)$.

B.1.2 Claim (2): Probability of payout

Using the insights from Dixit and Pindyck (1994), the dynamics of $P_p(c, X)$ are given by

$$P_p'(c)(rc + \mu) + \frac{\sigma^2}{2} P_p''(c) - \lambda \ P_p(c) = 0$$
$$\text{s.t. } P_p(0) = 0, \quad P_p(X) = \ 1.$$

The first boundary condition implies that when the controlled cash process is absorbed at zero, the firm liquidates and the payout probability is zero. The second boundary condition is obvious given that cash is paid out at X. The following lemma shows that greater bid-ask spreads are associated with larger payout probability.

Lemma 14 *For any* $\eta_1 > \eta_2$, $P_p(c, C_V(\eta_1)) \geq P_p(c, C_V(\eta_2))$.

Proof. By Lemma 11, $C_V(\eta_1) < C_V(\eta_2)$. To ease the notation throughout the proof, I define $X_1 \equiv C_V(\eta_1)$ and $X_2 \equiv C_V(\eta_2)$. Consider the function

$$h(c) = \ P_p(c, X_1) - P_p(c, X_2).$$

Because of the boundary conditions at zero and X_1, $h(0) = \ 0$ and $h(X_1) = 1 - P_p(c, X_2) > 0$. This means that the function is null at zero, and positive at C_V. Note that $h(c)$ cannot have neither a positive local maximum ($h(c) > 0$, $h'(c) = 0$, $h''(c) < 0$) nor a negative local minimum ($h(c) < 0$, $h'(c) = 0$, $h''(c) > 0$) on $[0, X_1]$, as otherwise the equation $h''(c)\frac{\sigma^2}{2} + h'(c)[rc + \mu] - \lambda h(c) = 0$ would not hold. Therefore, the function must be always positive and increasing over the relevant interval, and the claim follows. ■

The result below is a straightforward consequence of Lemma 14 and the fact that, in the absence of trading costs, $\eta = 0$ (or $\delta = 0$).

Corollary 15 *When trading the firm's stock is costly, the payout probability* P_p *is larger than in the benchmark case with no trading costs, i.e.* $P_p(c, C^*) < P_p(c, C_V)$.

These results can be extended for two parameters $\delta_1 > \delta_2$, as follows.

Corollary 16 *For any* $\delta_1 > \delta_2$, $P_p(c, C_V(\delta_1)) \geq P_p(c, C_V(\delta_2))$.

B.1.3 Claim (3): Probability of liquidation

I derive the results regarding the probability of liquidation $P_l(c, X)$, because the probability of external financing is just $P_f(c, X) = 1 - P_l(c, X)$. Using standard methods (see e.g., Dixit and Pyndick, 1994), the dynamics of $P_l(c, X)$ are given by

$$P_l'(c)(rc + \mu) + \frac{\sigma^2}{2} P_l''(c) - \lambda\, P_l(c) = 0 \tag{24}$$

$$\text{s.t. } P_l(0) = 1 \tag{25}$$

$$P_l'(X) = 0, \tag{26}$$

where the first boundary condition is given by the definition of P_l, while the second boundary condition is due to reflection at the payout threshold.

Now I prove that the probability of liquidation is higher when the firm's stocks are illiquid (i.e, the bid-ask spread associated with the firm stock is positive). To do so, I first prove that the probabilities $P_l(c, C^*)$ and $P_l(c, C_V)$ are decreasing and convex in c. In the following, I employ the generic function $P_l(c, X) \equiv P_l(c)$.

Lemma 17 *The probability* $P_l(c, X)$ *is decreasing and convex for any* $c \in [0, X]$.

Proof. As $P_l'(X) = 0$ and $P_l(X) \geq 0$, it must be that $P_l''(X) > 0$ for equation (24) to hold. Then, there exists a left neighbourhood of X, $[X - \epsilon, X]$ with $\epsilon > 0$, over which $P_l'(c) < 0$ and $P_l''(c) > 0$. Toward a contradiction, suppose that there exists some $c \in [0, X - \epsilon]$ where $P_l'(c) > 0$. Then, there should be a $\bar{C}$ such that $P_l'(\bar{C}) = 0$, while $P_l'(c) < 0$ for $c \in [\bar{C}, X]$. For any $c \in [\bar{C}, X]$ it must be that

$$P_l''(c) = \frac{2}{\sigma^2}\left[\lambda P_l(c) - P_l'(c)(rc + \mu)\right] > \frac{2}{\sigma^2}\, \lambda P_l(X) \; > \; 0\,.$$

Then, $P_l''(c) > 0$ for any $c \in [\bar{C}, X]$ means that $P_l'(c)$ is always increasing on $c \in [\bar{C}, X]$, contradicting $P_l'(\bar{C}) = P_l'(X) = 0$. The claim follows. ■

Now I prove that $P_l(c, C_V) \geq P_l(c, C^*)$.

Lemma 18 *For any* $\eta_1 > \eta_2$, $P_l(c, C_V(\eta_1)) \geq P_l(c, C_V(\eta_2))$.

Proof. By Lemma 11, $C_V(\eta_1) < C_V(\eta_2)$. To ease the notation throughout the proof, I define $X_1 \equiv C_V(\eta_1)$ and $X_2 \equiv C_V(\eta_2)$. By Lemma 17, the functions $P_l(c, X_1)$ and $P_l(c, X_2)$ are positive, decreasing and convex over the interval of definition. I define the auxiliary function

$$h(c) = P_l(c, X_1) - P_l(c, X_2).$$

Note that $h(c)$ cannot have neither a positive local maximum ($h(c) > 0$, $h'(c) = 0$, $h''(c) < 0$) nor a negative local minimum ($h(c) < 0$, $h'(c) = 0$, $h''(c) > 0$) on $[0, X_1]$, as otherwise the equation $h''(c)\frac{\sigma^2}{2} + h'(c)[rc + \mu] - \lambda h(c) = 0$ would not hold. In addition, $h(0) = 0$, and $h'(X_1) = -P_l'(c, X_2) > 0$ because of the boundary conditions at zero and at X_1. This means that the function is null at the origin, and increasing at C_V. Toward a contradiction, assume that $h(X_1)$ is negative. This would imply the existence of a negative local minimum, given that the function is null at zero and it is increasing at X_1. This cannot be the case as argued above, contradicting that $h(X_1) < 0$. Therefore, the function must be always positive, and the claim follows. ■

The result below is a straightforward consequence of Lemma 18 and the fact that, in the absence of trading costs, $\eta = 0$ (or $\delta = 0$).

Corollary 19 *When the bid-ask spread associated with the firm's stock is positive, the probability of liquidation P_l is larger than in the case in which the bid-ask spread is zero, i.e. $P_l(c, C^*) < P_l(c, C_V)$.*

B.1.4 Claim (4): Zero-NPV cost

I follow the same steps as in Appendix A.1 and exploit the results in Décamps and Villeneuve (2007) and HMM, namely, the growth option has a non-positive NPV if and only if $V(c) > V_+(c - I)$ for any $c \geq 0$.A straightforward modification of Lemma 8 confirms the claim. Notably, the zero-NPV cost satisfies $I_V \equiv V_+(C_{V+}) - C_{V+} - (V(C_V) - C_V)$, which gives equation (16) by calculations.

B.1.5 Claim (5): Firm value

Consider $\eta_1 > \eta_2$ and define $X_1 \equiv C_V(\eta_1)$ and $X_2 \equiv C_V(\eta_2)$. Consider the auxiliary function $h(c) = V(c; X_1, \eta_1) - V(c; X_2, \eta_2)$. Using equation (13), it satisfies the following dynamics

$$(\rho + \delta\eta_1)h(c) + \delta(\eta_1 - \eta_2)V_2(c; X_1, \eta_1) = (rc + \mu)\, h'(c) + \frac{\sigma^2}{2}h''(c) + \lambda[h(X_2) - h(c)]. \tag{27}$$

Because of the boundary condition at zero, $h(0) = 0$ and we have that $h'(X_2) = 0$ and $h'(X_1) = 1 - V'(c, X_2) < 0$ because of the boundary conditions at the thresholds X_1, X_2. As the function is non-increasing over $[X_1, X_2]$ and $h(0) = 0$, either the function is negative for any $c \in [0, X_2]$ or it has a positive local maximum over $[0, X_1)$ (at which $h() > 0$, $h'() = 0$, $h''() < 0$). At such maximum, the left-hand side of equation (27) would be positive, whereas the right-hand side would be negative (i.e., if this is a positive maximum, the last term on the right-hand side is negative). Thus, such a maximum cannot exist, meaning that h is decreasing and negative for any $c \in [0, X_2]$. The claim follows. ■

C Proof of the Results in Section 4.2

C.1 Proof of Proposition 3

I prove Proposition 3 claim by claim.[29] Notably, claims (2)-(4) importantly rely on the proof of claim (1) about the target cash threshold, from which I start.

[29] For simplicity, I focus on the case in which $\eta(c) < \chi$ for any $c \leq C_V$.

As in Appendix B, I express the function $V(c)$ as a function of X, which denotes the threshold satisfying $V'(X,X)-1=V''(X,X)=0$. By a straightforward modification of Lemma 9, it is possible to show that $V(c,X)$ is decreasing in X. I exploit the following Lemma.

Lemma 20 *For a given X and two given $\nu_1>\nu_2$, $V(c,X,\nu_2)>V(c,X,\nu_1)$ holds for any $c\in[0,X]$.*

Proof. I define the auxiliary function $h(c)=V(c,X,\nu_2)-V(c,X,\nu_1)$. At $c=X$, the function is positive as

$$h(X)=(rX+\mu-\zeta\omega)\left(\frac{1}{\rho+\delta\nu_2}-\frac{1}{\rho+\delta\nu_1}\right)>0 \tag{28}$$

given the assumption that $\nu_1>\nu_2$. Moreover, $h'(X)=h''(X)=0$. The function $h(c)$ satisfies the following dynamics:

$$(rc+\mu)h'(c)+\frac{\sigma^2}{2}h''(c)-(\rho+\lambda+\delta\nu_2)h(c)+\lambda h(X)=(\delta\nu_2-\delta\nu_1)V(c,X;\nu_1) \tag{29}$$

where the right-hand side is negative. Differentiating gives

$$(rc+\mu)h''(c)+\frac{\sigma^2}{2}h'''(c)-(\rho+\lambda+\delta\nu_2-r)h'(c)=(\delta\nu_2-\delta\nu_1)V'(c,X;\nu_1). \tag{30}$$

At X, using the boundary conditions, I get $\frac{\sigma^2}{2}h'''(X)=(\delta\nu_2-\delta\nu_1)<0$. This means that h'' is decreasing in a left neighborhood of X which, together with $h''(X)=0$, implies that $h''>0$ in such a neighborhood. In turn, this means that $h'(c)$ is increasing in such a neighborhood which, together with the boundary at X, implies that $h'(c)<0$ in such neighborhood. By equation (30), $h'(c)$ cannot have a negative local minimum (where $h'<0$, $h''=0$, $h'''>0$). As $h'(X)=0$ and h' is negative and increasing in a left neighborhood of X, it should be always negative for any $c<X$, so h is always decreasing. As it is positive at X, it is then positive for any $c<X$. ■

Exploiting the result above, I can prove the following.

Lemma 21 *For any $\nu_1>\nu_2$, then $C_V(\nu_1)<C_V(\nu_2)$*

Proof. The target cash thresholds are the unique solution to the boundary conditions $V(0,C_V(\nu_i);\nu_i)-\ell=0$. Exploiting Lemma (20), I have

$$V(0,C_V(\nu_1);\nu_2)-\ell>0=V(0,C_V(\nu_1);\nu_1)-\ell.$$

As V is decreasing in the payout threshold, this means that $C_V(\eta_1)<C_V(\eta_2)$ to get the equality $V(0,C_V(\nu_2);\nu_2)-\ell=0$. The claim follows. ■

Lemma 22 *For any $\omega_1>\omega_2$, $C_V(\omega_1)<C_V(\omega_2)$*

Proof. I start by proving that, for a given threshold X so that $V'(X)-1=V''(X)=0$ the inequality $V(c,X,\omega_2)>V(c,X,\omega_1)$ holds for any $c\in[0,X]$. To this end, I follow arguments as in Lemma 20 and define the auxiliary function $h(c)=V(c,X;\omega_2)-V(c,X;\omega_1)$. By calculations, it is possible to show that $h(X)=\zeta(\omega_1-\omega_2)/(\rho+\delta\nu)>0$ as $\omega_1>\omega_2$ by assumption. Moreover, $h'(X)=h''(X)=0$ by the boundary conditions and, using standard arguments, h satisfies $(\rho+\lambda+\delta\nu)h(c)=(rc+\mu)h'(c)+\frac{\sigma^2}{2}h''(c)+\lambda h(X)-\zeta(\omega_2-\omega_1)$, where the sum of the last two terms is positive—thus, $h(c)$ cannot have a negative local minimum over the interval $[0, X]$ (where $h < 0$, $h' = 0$, $h'' > 0$) as the equation would not hold. Differentiating the above equation, I obtain $(\rho + \lambda + \delta\nu - r)h'(c) = (rc + \mu)h''(c) + \frac{\sigma^2}{2}h'''(c)$ so that $h'''(X) = 0$ (and, continuing differentiating this equation, I can show that all the subsequent derivatives are zero). Thus, h is positive for any $c \in [0, X]$. Using arguments similar to Lemma 21, the claim follows. ■

Exploiting that C_V is monotonic in ν and ω as just shown, claims (2) and (3) about the impact of ν and ω on the probability of liquidation and of payout follow by using steps similar to those used in Appendices B.1.3 and B.1.2.

Lastly, I show that firm value is monotonic in ν and ω (claim (4)). Consider first the impact of ν. Define $\nu_1 > \nu_2$. I define $X_1 \equiv C_V(\nu_1)$ and $X_2 \equiv C_V(\nu_2)$ and $X_1 < X_2$ as proved in claim (1). I define the auxiliary function: $h(c) = V(c, X_1, \nu_1) - V(c, X_2, \nu_2)$. Using equation (19), it satisfies the following dynamics

$$(\rho + \delta\nu_1)h(c) + \delta(\nu_1 - \nu_2)V(c, X_2, \nu_2) = (rc + \mu)\, h'(c) + \frac{\sigma^2}{2}h''(c) + \lambda[h(X_2) - h(c)].$$

Because of the boundary condition at zero, $h(0) = 0$ and we have that $h'(X_2) = 0$ and $h'(X_1) = 1 - V'(c, X_2) < 0$ because of Lemma 6. Thus, either the function is negative for any $c \in [0, X_2]$ or it has a positive local maximum (at which $h > 0$, $h' = 0$, $h'' < 0$). At such maximum, the left-hand side of equation (31) would be positive, whereas the right-hand side would be negative (i.e., if this is a positive maximum, the last term on the right-hand side is negative). Thus, such a maximum cannot exist, meaning that h is negative for any $c \in [0, X_2]$.

Consider now $\omega_1 > \omega_2$. As above, define $X_1 \equiv C_V(\omega_1)$ and $X_2 \equiv C_V(\omega_2)$. and $X_1 < X_2$ by claim (1). I define the auxiliary function: $h(c) = V(c, X_1) - V(c, X_2)$, which satisfies:

$$(\rho + \delta\nu)h(c) = (rc + \mu)\, h'(c) + \frac{\sigma^2}{2}h''(c) + \lambda[h(X_2) - h(c)] - \zeta(\omega_1 - \omega_2). \tag{31}$$

As above, because of the boundary condition at zero, $h(0) = 0$ and we have that $h'(X_2) = 0$ and $h'(X_1) = 1 - V'(c, X_2) < 0$. In this case too, a positive maximum cannot exists: The left-hand side of equation (31) would be positive, whereas the right-hand side would be negative (i.e., if this is a positive maximum, the third term on the right-hand side is negative, and the last term too given the assumption that $\omega_1 > \omega_2$). Thus, such a maximum cannot exist, and the claim follows. ■

C.2 Proof of Proposition 4

The Proposition exploits the expression for the equilibrium bid-ask spread (see equation (8)) and the concavity of firm value proved in Appendix A. The claim follows. ■

D Proof of the Results in Section 4.4

Consider the zero-profit condition of trading firms under the DMM contract:

$$\delta\Big\{-(1-\eta)\, V(c;\eta) + V(c;\eta)(1-\nu)\Big\} + \Gamma = \zeta\omega.$$

This condition differs from the baseline environment without DMM in that the trading firms receive the periodic flow payment Γ. Calculations then give the bid-ask spread reported in equation (17).

Next, I prove Proposition 5.

Proof. Comparing equation (18) with equation (19) illustrates that the two equations differ for the additional terms $-\Gamma V'(c) + \Gamma$ on the right-hand side. The sum of this two terms can be rewritten as $\Gamma(1 - V'(c))$. As $V'(c) \geq 1$ for any $c < C_V$, the expression $\Gamma(1 - V'(c))$ is strictly negative for any $c < C_V$, which implies that it is suboptimal for a financially constrained firm to enter the DMM contract. ■

To focus on the effects on firm value (which are the prime goal of this extension), the analysis does not provide additional assumptions to explain how to pin down the periodic fee paid by the firm to the DMM. For example, a fee that would warrant that the maximum bid-ask spread is attained at $c = 0$, and $\eta(c) < \bar{\eta}$ for any positive c, would satisfy:

$$\delta \left[-(1 - \bar{\eta})V(0) + V(0)(1 - \nu) \right] + \Gamma = \zeta\omega, \tag{32}$$

which gives $\Gamma^* = \zeta\omega - \delta(\bar{\eta} - \nu)V(0)$. While other assumptions could be considered, it is worth emphasizing that Proposition 5 holds for any Γ—i.e., it is *not* contingent on a particular specification.

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Symbol	Description	Value
	FIRM	
ρ	Risk-free rate	0.02
r	Return on cash	0.01
μ	Cash flow drift	0.05
μ_+	Post-investment cash flow drift	0.06
σ	Cash flow volatility	0.12
ϕ	Recovery rate in liquidation	0.55
λ	Arrival rate of financing opportunities	0.75
	STOCK TRANSACTIONS	
δ	Arrival rate of liquidity shocks	0.700
χ	Shocked shareholders' holding cost	0.020
ν	Order-processing cost	0.002
ζ	Arrival rate of trading opportunities in other markets	0.900
ω	Mean payoff from trading opportunities in other markets	0.006
$\bar{\eta}$	DMM's bid-ask cap	0.007

TABLE 1: BENCHMARK PARAMETERS.

	Target cash level	Liquidation probability	Payout probability	Zero-NPV cost	Firm value	Bid-ask spread (basis points)
$\nu = 0.001$	-7.00%	0.97%	1.56%	-4.20%	-13.45%	42
$\nu = 0.002$	-9.60%	1.37%	2.19%	-7.55%	-16.53%	53
$\nu = 0.003$	-12.29%	1.82%	2.86%	-10.73%	-19.41%	65
$\nu = 0.004$	-15.09%	2.31%	3.59%	-13.77%	-22.13%	76
$\nu = 0.005$	-18.06%	2.87%	4.39%	-16.71%	-24.69%	87
$\omega = 0.004$	-7.62%	1.07%	1.71%	-7.19%	-13.31%	41
$\omega = 0.006$	-9.60%	1.37%	2.19%	-7.55%	-16.53%	53
$\omega = 0.008$	-11.94%	1.76%	2.77%	-8.00%	-19.77%	66
$\omega = 0.010$	-14.76%	2.25%	3.50%	-8.61%	-23.07%	80
$\omega = 0.012$	-18.31%	2.92%	4.46%	-9.47%	-26.43%	96

TABLE 2: ENDOGENOUS BID-ASK SPREAD AND CORPORATE OUTCOMES.

The table reports the change in corporate policies and outcomes in the setup with endogenous bid-ask spread compared to a benchmark environment with zero bid-ask spread (and, thus, perfect stock liquidity). Namely, the table shows the change in the target cash level, in the probability of liquidation, in the probability of payout, in the zero-NPV investment cost, and in firm value (to fix ideas, calculated at the target cash level) as well as the bid-ask spread (also calculated at the target cash level) for different values of the order-processing cost ν (top panel) and of the expected payoff from outside opportunities ω (bottom panel).

	$C_V/2$	$C_V/4$	$C_V/8$
Varying ν			
$\nu = 0.001$	2.60%	16.20%	40.31%
$\nu = 0.002$	2.89%	17.05%	41.36%
$\nu = 0.003$	3.22%	17.98%	42.46%
$\nu = 0.004$	3.60%	19.00%	43.64%
$\nu = 0.005$	4.06%	20.14%	44.93%
Varying ω			
$\omega = 0.004$	2.67%	16.39%	40.56%
$\omega = 0.006$	2.89%	17.05%	41.36%
$\omega = 0.008$	3.17%	17.85%	42.31%
$\omega = 0.010$	3.56%	18.87%	43.50%
$\omega = 0.012$	4.11%	20.24%	45.04%
Perfect liquidity	1.97%	14.11%	37.64%

TABLE 3: THE FIRM'S PROBABILITY OF FORCED LIQUIDATION IN THE PRESENCE OF STOCK MARKET ILLIQUIDITY.

The table reports the firm's probability of liquidation at different levels of cash reserves (i.e., at $C_V/2$, $C_V/4$, and $C_V/8$) when varying the order-processing cost ν and the expected payoff from outside opportunities ω. The bottom line reports the probability of liquidation when the firm's stock is perfectly liquid (i.e., when the bid-ask spread is zero).

	Direct effect	With indirect effect
Baseline $\omega = 0.006$		
$\Delta\omega = +0.002$	+11	+13
$\Delta\omega = +0.003$	+17	+20
$\Delta\omega = +0.004$	+22	+27
$\Delta\omega = +0.005$	+28	+34
$\Delta\omega = +0.006$	+33	+42
$\Delta\omega = +0.007$	+39	+51
$\Delta\omega = +0.008$	+44	+59
$\Delta\omega = +0.009$	+50	+68
$\Delta\omega = +0.010$	+56	+78
Baseline $\nu = 0.002$		
$\Delta\nu = +0.0015$	+15	+17
$\Delta\nu = +0.0020$	+20	+22
$\Delta\nu = +0.0025$	+25	+28
$\Delta\nu = +0.0030$	+30	+34
$\Delta\nu = +0.0035$	+35	+39
$\Delta\nu = +0.0040$	+40	+45
$\Delta\nu = +0.0045$	+45	+50
$\Delta\nu = +0.0050$	+50	+56
$\Delta\nu = +0.0055$	+55	+62

TABLE 4: DIRECT AND INDIRECT EFFECTS OF SHOCKS TO LIQUIDITY PROVISION.

The table reports the basis point increase in the bid-ask spread following a rise in ω (top panel, where I denote the magnitude of the increase by $\Delta\omega$) or in ν (bottom panel, where I denote the magnitude of the increase by $\Delta\nu$) compared to their baseline value. I report the direct effect in the middle column, whereas I report the whole effect (including the indirect effect on corporate policies and value) in the right column. To fix ideas, I calculate the bid-ask at the target cash level.

	Bid-ask spread	Firm value (illiquidity)	Firm value (benchmark)	Value drop (%)
Baseline				
$\mu = 0.03$	77	1.348	1.770	-23.9%
$\mu = 0.04$	62	1.840	2.274	-19.1%
$\mu = 0.05$	53	2.315	2.773	-16.5%
$\mu = 0.06$	48	2.783	3.269	-14.9%
$\mu = 0.07$	44	3.248	3.763	-13.7%
$\mu = 0.08$	41	3.712	4.257	-12.8%
$\mu = 0.09$	38	4.174	4.750	-12.1%
$\mu = 0.10$	37	4.637	5.243	-11.6%
Higher ν and ω				
$\mu = 0.03$	110	1.208	1.770	-31.8%
$\mu = 0.04$	85	1.712	2.274	-24.7%
$\mu = 0.05$	72	2.185	2.773	-21.2%
$\mu = 0.06$	64	2.648	3.269	-19.0%
$\mu = 0.07$	58	3.108	3.763	-17.4%
$\mu = 0.08$	54	3.565	4.257	-16.3%
$\mu = 0.09$	51	4.021	4.750	-15.4%
$\mu = 0.10$	48	4.476	5.243	-14.6%

Table 5: Bid-ask spread and firm value in the cross section.

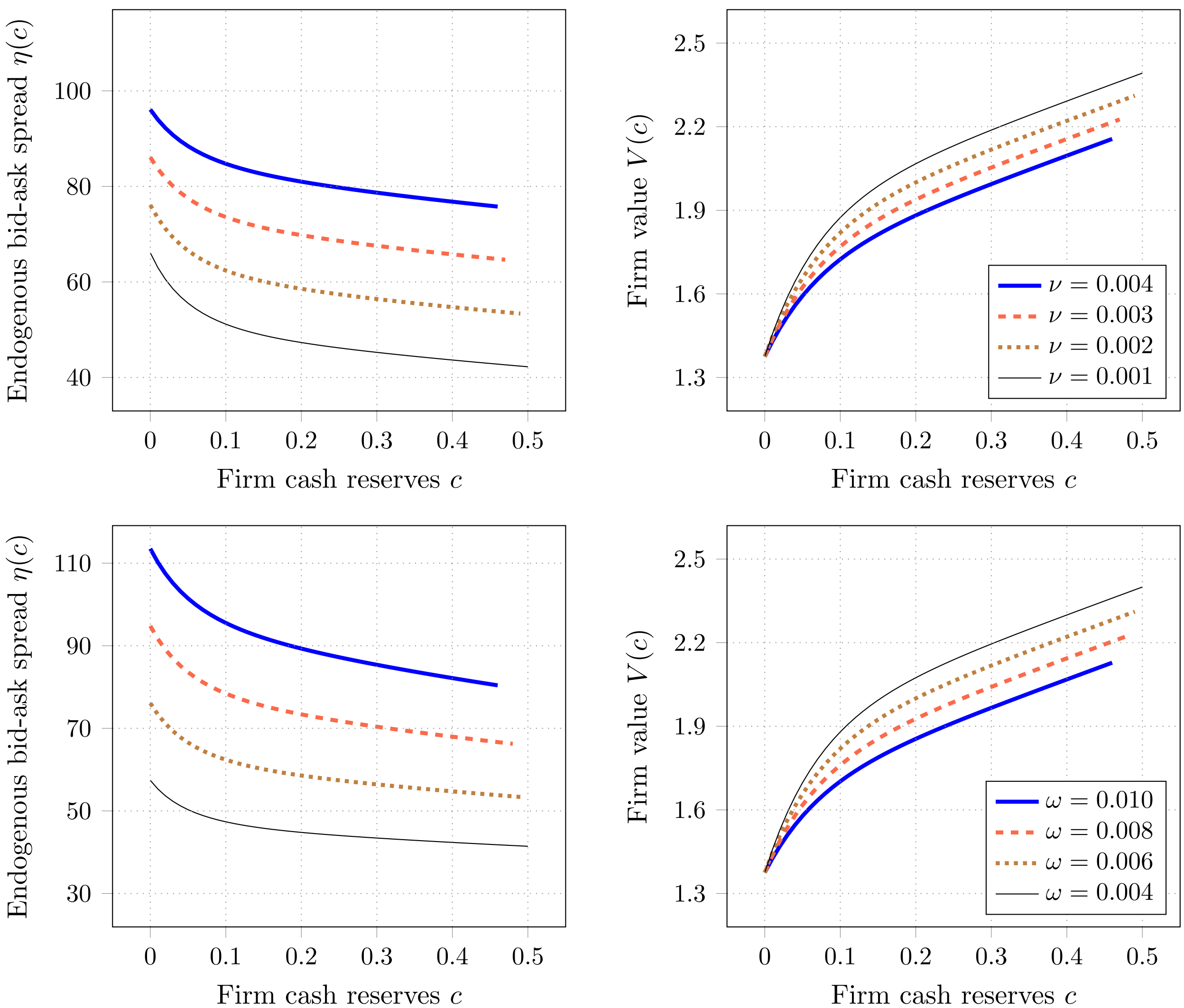
The table reports the bid-ask spread (second column), the firm value when the bid-ask spread is endogenous (third column), the firm value when stock liquidity is perfect (i.e., the bid-ask spread is zero, fourth column), and the drop in firm value due to endogenous illiquidity (fifth column) when varying the parameter μ, serving as a gauge of size. To fix ideas, we calculate these quantities when the firm holds its target cash level. The top panel focuses on the baseline parameterization for ν and ω (as reported in Table 1), whereas the lower panel assumes more costly liquidity provision (by assuming that $\nu = 0.0025$ and $\omega = 0.008$).

	Target cash level			Liquidation probability		
	C^*	C_V	Wedge	$\bar{P}_l(C^*)$	$\bar{P}_l(C_V)$	Wedge
$\lambda = 0.25$	0.638	0.566	-11.37%	15.08%	17.17%	2.09%
$\lambda = 0.50$	0.584	0.524	-10.29%	13.60%	15.20%	1.60%
$\lambda = 0.75$	0.546	0.493	-9.60%	12.78%	14.15%	1.37%
$\lambda = 1.00$	0.516	0.469	-9.12%	12.22%	13.46%	1.24%
$\lambda = 1.25$	0.493	0.450	-8.75%	11.81%	12.95%	1.14%
$\lambda = 1.50$	0.473	0.433	-8.45%	11.49%	12.55%	1.06%
$\sigma = 0.08$	0.347	0.319	-7.82%	11.48%	12.47%	0.99%
$\sigma = 0.10$	0.448	0.409	-8.71%	12.18%	13.36%	1.18%
$\sigma = 0.12$	0.546	0.493	-9.60%	12.78%	14.15%	1.37%
$\sigma = 0.14$	0.640	0.573	-10.52%	13.31%	14.90%	1.59%
$\sigma = 0.16$	0.731	0.647	-11.47%	13.80%	15.61%	1.81%
$\sigma = 0.18$	0.818	0.716	-12.46%	14.26%	16.31%	2.06%

Table 6: Cash hoarding and forced liquidation with endogenous bid-ask spread.

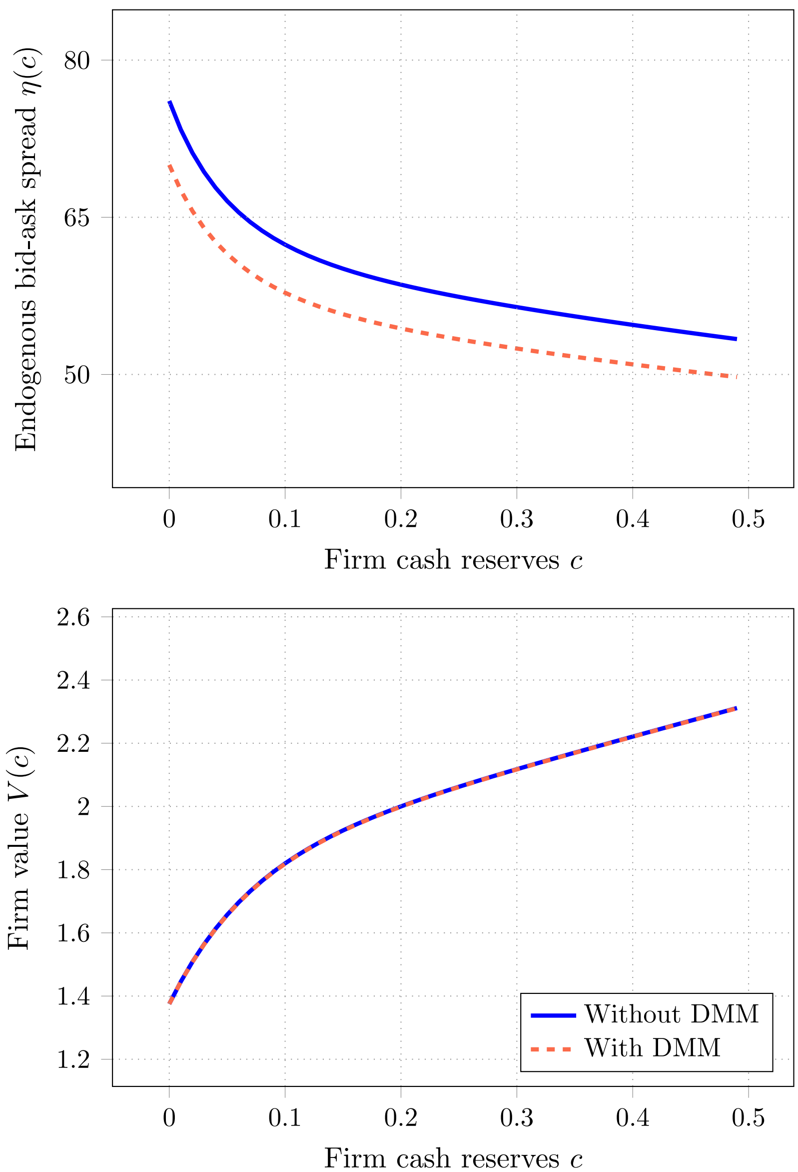
The table reports the target cash level and the average liquidation probability in the benchmark environment in which stock liquidity is perfect (i.e., the bid-ask spread is zero, second and fifth columns), in the baseline parameterization with endogenous bid-ask spread (third and sixth columns), and the wedge between the two environments (fourth and seventh column) when varying the firm's access to external financing (λ) and cash flow volatility (σ).

Figure 1: Endogenous bid-ask spread and firm value.



The figure shows the endogenous bid-ask spread (in basis points, left panels) as well as firm value (right panels) as a function of the firm cash reserves c when varying the order-processing cost ν (top panels) and the expected payoff from outside opportunities ω (bottom panels).

Figure 2: Firm-funded designated market makers (DMM).



The figure shows the endogenous bid-ask spread (in basis points) as well as firm value as a function of the firm cash reserves c. The solid blue line depicts the case in which the firm does not enter the DMM contract, whereas the dashed red line depicts the environment in which the firm enters the DMM contract requiring to correspond a periodic fee.