The Real Effects of Financing and Trading Frictions*

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Abstract

I develop a model that studies the interplay between investors' trading frictions and firms' financial constraints as well as the ensuing effects on corporate policies. The model shows that bid-ask spreads increase not only the cost of external financing but also the cost of internal funds, leading to smaller cash reserves and larger payouts. Thus, firm's financial constraints tighten, liquidation risk increases, investment decreases, and firm value declines. These outcomes are reinforced when internalized by liquidity providers, giving rise to a feedback that makes the bid-ask spread wider and amplifies its detrimental effects on firm financial constraints and value.

Keywords: Financial constraints, transaction costs, real effects of financial markets, small firms

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1 Introduction

Corporate financial constraints and investors' trading frictions appear to go hand in hand in the cross-section of firms. Large firms enjoy an easy access to external financing, and their stocks are very liquid. At the other side of the spectrum, small firms are largely financially constrained, and their stocks are relatively illiquid. Indeed, small firms typically face delays and costs when raising fresh funds, an issue that has spurred the creation of an ad-hoc committee within the U.S. Securities and Exchange Commission (SEC).[1] Moreover, their stocks are characterized by non-negligible bid-ask spreads, low trading volume, and other microstructure frictions (e.g., Hou et al., 2016; Novy-Marx and Velikov, 2016; Chordia et al., 2011). As small firms represent more than 80% of U.S. firms over the past forty years (Hou et al., 2020), understanding the frictions affecting their performance is of utmost importance.

This paper develops a dynamic model that shows that trading frictions and financial constraints are deeply related and investigates their real effects. The model studies a firm that has assets in place—which generate a stochastic flow of revenues—and a growth option. The firm faces uncertainty in its ability to raise external financing, as small firms do in the real world. The model also assumes that shareholders and liquidity providers face frictions when trading the firm stocks, consistent with the frictions faced by small firms' investors. The model shows that trading frictions affect corporate financial and investment decisions. In turn, these corporate policies and value impact the firm's bid-ask spread, then shedding light on a novel feedback between financial markets and corporations through which investors' trading frictions and the firm's financial constraints reinforce each other.

To disentangle the forces at play, I start by examining an environment in which shareholders face a constant bid-ask spread when trading the firm stock. To compensate for the ensuing trading losses, the model shows that investors require a larger return to hold the stock, similar to Amihud and Mendelson (1986) and Acharya and Pedersen (2005). As a

[1] Small firms are typically less known and more vulnerable to capital market imperfections. In contrast, large, established firms are more likely to have continued relations with financial institution and are less subject to asymmetric information. The U.S. SEC Advisory Committee in Small and Emerging Companies pointed out that small firms often struggle to attract capital, see https://www.sec.gov/spotlight/advisory-committee-on-small-and-emerging-companies.shtml.

result, the cost of external equity increases and, all else equal, should incentivize the firm to keep more cash. At the same time, however, the greater return required by the investors also generates an offsetting strength: it expands the wedge between the firm's cost of capital and the return on cash, then increasing its opportunity cost. On net, I demonstrate that firms whose stocks are traded at larger bid-ask spreads are more financially constrained. As a result, these firms are more exposed to forced liquidations, as they are less likely to raise external financing and keep smaller precautionary cash reserves. Moreover, these firms face a severe underinvestment problem, as the additional return required by the investors erodes the profitability of investment opportunities. Overall, firm value decreases.

I next relax the assumption that the bid-ask spread is constant and exogenous. To do so, I put more structure into the model and realistically assume that liquidity providers are competitive and face participation frictions (i.e., participation fees and funding constraints). In this richer setup, the bid-ask spread not only affects but also reflects corporate policies, giving rise to a feedback effect. As shown above, the bid-ask spread reduces firm value. When liquidity provision is endogenous, such a decrease in firm value feeds back into the bid-ask spread by leading liquidity providers to extract larger rents from shocked shareholders as a proportion of the value of their claim to cover their participation and funding costs. As a result, the bid-ask spread widens, and its detrimental effects on corporate policies and value strengthen—in particular, financial constraints tighten, and the probability of forced liquidations increases. That is, this feedback not only makes trading more costly for shareholders but also exacerbates the detrimental effects of the bid-ask spread on firm value.

The model delivers a rich set of predictions. First, it characterizes corporate policies and outcomes for those firms, such as small ones, whose stocks are relatively illiquid. The model predicts that these firms should face severe financial constraints because of their larger costs of external *and* internal equity, which reduce the probability of external financing and the size of the firm's precautionary cash reserves. As a result, these firms face higher liquidation risk, consistent with Brogaard et al. (2017). Notwithstanding these constraints, these firms should exhibit larger payouts in the cross-section to compensate investors for trading frictions, consistent with Banerjee et al. (2007). These firms should also face an underinvestment problem, consistent with Campello et al. (2014) and Amihud and Levi (2019).

Second, the model supports the evidence that liquidity supply has notable effects on bid-ask spreads—especially for small-capitalization stocks, as illustrated by Anand et al. (2013) and Aragon and Strahan (2012)—as well as on corporate financing and investment (see, e.g., Goldberg, 2020). Indeed, the model illustrates that frictions faced by liquidity providers are passed on to the firm's investors by leading to larger bid-ask spreads which, in turn, affect corporate outcomes and value. The model also shows that shocks that affect the participation of liquidity providers (e.g., making market presence more costly) have an amplified effect on a firm's bid-ask spread when reflected into firm value. While leading to a direct increase in the cost of trading for investors, these shocks also decrease firm value which, through the feedback effect illustrated in the paper, calls for a further increase in the cost of trading. Notably, this feedback effect implies that trading frictions and financial constraints reinforce each other, then rationalizing the observation that they are disproportionately larger for small firms vis-à-vis large firms.[2]

The model predictions are robust to a battery of alternative assumptions. First, I show that the results are robust to modeling financing frictions as issuance costs as well as to allowing the firm to borrow from bank credit lines as an additional source of liquidity. Furthermore, I model dynamic investment as in the neoclassical q framework of Bolton et al. (2011), then allowing for cash and capital accumulation. This setup shows that if financial constraints are sufficiently lax so that the firm engages in positive investment, the investment rate decreases with the magnitude of the bid-ask spread—i.e., the bid-ask spread engenders underinvestment. Finally, I alternatively endogenize liquidity provision by allowing the mass of trading firms following the stock to vary over time. In this setting, the feedback effect between liquidity provision and corporate policies continues to hold.

Notably, the model provides a tractable framework to assess regulation targeting equity markets from a corporate perspective. In particular, academics and policy-makers have

[2]Indeed, the feedback effect should be less relevant for large firms, characterized by easy access to outside financing, and whose stocks are traded relatively more often and at small bid-ask spreads.

been questioning whether market forces guarantee enough liquidity provision, especially for small-capitalization stocks. Several stock markets have then contemplated the possibility for firms to engage a designated market maker (DMM) to maintain the bid-ask spread below an agreed-upon level. DMM contracts can help decrease the firm's cost of capital by capping investors' trading costs, but bind the firm to correspond a fee to the DMM. On net, an application of the model accommodating DMM contracts suggests that small, financially constrained firms would hardly find this contractual agreement value-enhancing.

Related literature This paper relates to the literature showing that frictions affecting stock trading impact corporate policies and outcomes. Fang et al. (2009) show that firms with liquid stocks are more valuable. Campello et al. (2014) find that stock liquidity improves corporate investment and value. Nyborg and Wang (2021) show that stock liquidity increases a firm's propensity to hold cash. Banerjee et al. (2007) reveal that firms with illiquid stocks pay out more dividends. Brogaard et al. (2017) find that stock liquidity reduces firms' bankruptcy risk. This paper develops a model that rationalizes these empirical findings into a unified framework. It also shows that these effects are amplified when liquidity providers internalize the negative effect of bid-ask spreads on firm value.

Since the 2007–2009 financial crisis, there has been a large interest in understanding the effects of liquidity providers' funding liquidity on market liquidity. Empirical works show that frictions faced by intermediaries affect the liquidity of their traded stocks, for instance, Anand et al. (2013); Hameed et al. (2010); Aragon and Strahan (2012); Comerton-Forde et al. (2010). In the meanwhile, a theoretical literature has developed to understand the dynamics of liquidity demand and supply and their impact on asset prices, e.g., Budish et al. (2015) or Huang and Wang (2010, 2009). These models abstract from informational frictions but, rather, focus on financial constraints and margin requirements faced by market participants (like intermediaries, broker-dealers, and other liquidity providers). A key assumption in these models is that the security traded by market participants promises a constant (exogenous) flow of dividends. In contrast, the current paper endogenizes the dividend flow associated with the traded security by investigating the optimal corporate policies of the issuing firm.

The paper is also related to the theoretical literature modeling endogenous feedback effects. Brunnermeier and Pedersen (2009) shows that there is a two-ways link between an asset's market liquidity and traders' funding liquidity. Traders provide market liquidity, which in turn depends on their funding ability. Because of margin requirements, traders' funding depends on the assets' market liquidity. The current paper instead focuses on the relation between the funding liquidity of a given firm and the market liquidity of its stocks. It shows that firm financing frictions and stock illiquidity reinforce each other, then providing a rationale as to why the degree of financial constraints and stock market illiquidity are disproportionately greater for small stocks vis-à-vis large firms. Another related paper is He and Milbradt (2014), who endogenize *bond* illiquidity into a Leland-type model of endogenous default, in which shareholders can inject fresh equity at no cost. He and Milbradt provide a decomposition of credit spreads into a default and a liquidity component, then matching several cross-sectional patterns of bid-ask spreads and credit spreads. Conversely, the present paper focuses on *stock* illiquidity and builds on the strand of dynamic corporate finance models with financing frictions, in which shareholders face costs or uncertainty in their ability to raise additional financing. This paper can reproduce and explain several documented effects of stock illiquidity on real and financial corporate policies (as illustrated above) and rationalizes the observation that frictions affecting liquidity supply importantly impact corporate decisions (see, e.g., Goldberg, 2020) and bid-ask spreads, especially for small firms (as illustrated by Anand et al., 2013; Aragon and Strahan, 2012).

Finally, this paper contributes to the strand of dynamic corporate finance models with financing frictions, including Bolton et al. (2011, henceforth BCW); Décamps et al. (2011, henceforth DMRV); Hugonnier et al. (2015, henceforth HMM); Malamud and Zucchi (2019); or Della Seta et al. (2020). These papers show that financing frictions, such as costs or uncertainty in raising external funds, should increase a firm's propensity to keep precautionary reserves. While these extant papers impose an exogenous cost of holding cash, the current model shows that this cost can arise endogenously when accounting for trading frictions faced by firm shareholders. Notably, it illustrates that trading frictions impact both the cost of internal and external financing, then affecting corporate policies and value.

The paper proceeds as follows. Section 2 describes the model. Section 3 analyzes the effects of a constant and exogenous bid-ask spread on corporate policies. Section 4 endogenizes the bid-ask spread, singling out a feedback effect between financial markets and corporations. Section 5 assesses the robustness of the model predictions to alternative modeling assumptions. Section 6 concludes. Proofs are gathered in the Appendix.

2 The Model

Time is continuous, and uncertainty is modeled by a probability space $(\Omega, \mathcal{F}, P)$ equipped with a filtration $(\mathcal{F}_t)_{t\geq 0}$. Agents are risk-neutral and discount cash flows at rate $\rho > 0$.

The Firm I consider a small firm operating a set of assets in place, which generate a continuous and stochastic flow of revenues. The flow of revenues is modeled as an arithmetic Brownian motion, $(Y_t)_{t\geq 0}$, whose dynamics evolve as

$$dY_t = \mu dt + \sigma dZ_t. \tag{1}$$

The parameters μ and σ are strictly positive and represent the mean and volatility of corporate revenues, and $(Z_t)_{t\geq 0}$ is a standard Brownian motion. The firm has access to a growth option that has the potential to increase its income stream from dY_t to $dY_t^+ = dY_t + (\mu_+ - \mu)dt$, $\mu_+ > \mu$, by paying a lump-sum cost $I > 0$. That is, the cash flow drift can assume two values $\mu_i = \{\mu, \mu_+\}$. Investment is assumed to be irreversible.

The cash flow process in equation (1) implies that the firm can make operating profits and losses. If capital supply was perfectly elastic, operating losses could be covered by raising outside financing immediately and at no cost. In practice, small firms face financing frictions, such as uncertainty or costs in raising funds. I model this capital supply uncertainty by assuming that the firm raises new funds at the jump times of a Poisson process, $(N_t)_{t\geq 0}$, with intensity λ, as in HMM. That is, if the firm decides to raise outside funds, the expected financing lag is $1/\lambda$ periods. If $\lambda \to 0$, the firm cannot raise external funds at all (equivalently, it takes an infinite waiting period to raise fresh funds upon searching) and relies on cash reserves to cover operating losses. If $\lambda \to \infty$, the waiting time upon searching for external funds is zero—i.e., the firm has access to outside financing at no delays. Notably, as shown in the paper, the discount on newly-issued equity is related to trading frictions faced by firms' investors. In Section 5.1, I show that the specific assumptions regarding the firm's financing frictions are without loss of generality—the main results of the paper continue to hold if financing frictions are modeled as issuance costs, as in DMRV or BCW.

Because capital supply is uncertain, the firm has incentives to retain earnings in cash reserves. I denote by $(C_t)_{t\geq 0}$ the firm's cash reserves at any t. Cash reserves earn a constant rate, $r \leq \rho$. Whenever $r < \rho$, keeping cash entails an opportunity cost.[3] In contrast with extant cash holdings models—in which the strict inequality $r < \rho$ is needed to depart from the corner solution featuring firms piling infinite cash reserves—I allow for the $r = \rho$ case. The cash reserves process satisfies:[4]

$$dC_t = \ rC_t dt + \mu_i dt + \sigma dZ_t - dD_t + f_t dN_t. \tag{2}$$

$dD_t \geq 0$ represents the instantaneous flow of payouts at time t. $f_t \geq 0$ denotes the instantaneous inflow of funds when financing opportunities arise, in which case management stores the proceeds in the cash reserves. This assumption is consistent with the strong, positive correlation between equity issues and cash accumulation documented by McLean (2011) or Eisfeldt and Muir (2016). Notably, D and f are endogenous. Equation (2) implies that the firm's cash reserves increase with external financing, retained earnings, and the interest earned on cash, whereas they decrease with payouts and operating losses.

Management can distribute cash and liquidate the firm's assets at any time. Yet, liqui-

[3]This cost can be interpreted as a free cash flow problem (Jensen, 1986) or as tax disadvantages (Graham, 2000).

[4]When investment occurs (meaning that the cash flow drift goes from μ to μ_+), the cost I is financed either with cash or external financing. Because the paper focuses on the decision of whether or not to invest (rather than on the investment timing), I do not explicitly model the outflow I when the growth option is exercised (which could be financed with outside funds).

dation is inefficient, as the recovery value of assets, denoted by ℓ, is smaller than the firm's first best, μ_i/ρ. These costs erode a fraction, $1 - \phi \in (0, 1]$, of the firm's first best, so the liquidation value is $\ell = \phi\mu_i/\rho$. I denote by τ the endogenous time of liquidation.

Transacting the Firm Stocks The key departure from previous dynamic corporate finance models with financing frictions is the explicit consideration of investors' stock transactions and the costs thereof. There are two types of risk-neutral traders: investors (who may buy, hold, and eventually sell the stock) and trading firms (or liquidity providers, which ease investors' trading).

Investors are ex-ante identical and infinitely lived. Each of them has measure zero and cannot short sell. Investors can be hit by liquidity shocks. As in previous contributions (e.g., Duffie et al., 2005; Lagos and Rocheteau, 2007; Bessembinder et al., 2015), liquidity shocks trigger a sudden need for liquidity that reduces the subjective valuation of the asset by a fraction χ.[5] Thus, χ can be interpreted as the opportunity cost of being locked into an undesired asset position—e.g., because of take-it-or-leave-it investment opportunities or unpredictable financing needs. The liquidity shock vanishes once the shocked investor either sells his stock or bears the loss χ. Liquidity shocks are independent across investors and occur at the jump times of a Poisson process with intensity $\delta > 0$. In turn, non-liquidity-shocked shareholders have no immediate need to trade and, thus, are indifferent between keeping the stock or selling it at its fundamental value.[6]

Trading firms are agents who maintain an active presence and provide liquidity in the market for the stock.[7] They have no intrinsic demand to buy or sell the firm's assets and are not subject to liquidity shocks. Trading firms pay a fixed flow cost as long as they are active in the market of the stock, denoted by γ, which can be interpreted as the cost of

[5] As in Bessembinder et al. (2015), the cost of liquidity shocks is proportional to the fundamental value of the asset held by the investor.

[6] Following previous contributions (see, e.g., He and Milbradt (2014)), I assume that the mass of non-liquidity-shocked investors is larger than that of liquidity-shocked shareholders, without loss of generality.

[7] Trading firms can be interpreted as high frequency traders, market makers, or algorithmic traders as in Budish et al. (2015), or simply as agents maintaining a constant market presence such as trading desks and hedge funds, as in Huang and Wang (2010).

monitoring and processing market movements. Moreover, I assume that liquidity providers are financially constrained. Their cost of funding erodes a fraction κ of their gross gain from liquidity provision. Following previous works, I assume that the market for liquidity provision is competitive, so that the equilibrium bid-ask spread is determined by the zero-profit condition.[8] For simplicity, the size of the trading firm sector is normalized to one. In Section 5.4, I relax this assumption and endogenize the mass of active trading firms.

Trading firms post bid and ask quotes. On the ask side, they trade with non-liquidity-shocked investors, who are indifferent between staying out of the market or buying the stock at its fundamental value (i.e., they do not have an immediate need to trade). As a result, the gain to trading firms on this side of the transaction is null. On the bid side, trading firms transact with shocked shareholders. Because shocked shareholders value the asset at a discount χ, trading firms can extract surplus from this side of transactions. The gain from transacting with shocked shareholders is denoted by $\eta \geq \kappa$—i.e., η represents the difference between the ask and the bid price (i.e., the bid-ask spread). This quantity is taken as exogenous in Section 3, and then derived endogenously in Section 4.

Equilibrium Firm management maximizes equity value. Namely, cash retention and payout (D), financing (f), liquidation (τ), and investment (I) are set to maximize:

$$V(c) = \sup_{(D,f,\tau,I)} \mathbb{E}\left[\int_0^{\tau} e^{-\rho t}\left(dD_t - f_t\, dN_t - dB_t\right) + e^{-\rho\tau}\ell\right], \tag{3}$$

which represents the expected present value of payouts net of outside financing (the first term) plus the liquidation value of assets (the second term). The expected flow of payouts to shareholders is drained by trading costs, whose cumulative process is denoted by B_t and is given by the sum of bid-ask spreads borne by shocked shareholders. In equilibrium, the

[8]This assumption is consistent with works in which competition among liquidity providers drives the surplus from trade to zero, e.g., among others Lagos and Rocheteau (2007) or He and Milbradt (2014).

bid-ask spread η is determined by the zero-profit condition, which is given by:

$$\text{Expected net gain from intermediating} = \text{Participation costs} \quad s.t. \quad \eta \leq \chi \tag{4}$$

on any time interval dt. The expected net gain from intermediating on any time interval is the difference between the price at which trading firms sell the stock to non-liquidity-shocked investors and the price at which they buy the stock from liquidity-shocked shareholders, net of the funding costs associated with such transaction. Notably, the equilibrium bid-ask spread η cannot exceed χ, otherwise shocked shareholders would be better off holding the stock instead of selling it to trading firms. Because the bid-ask spread is set endogenously in equilibrium, it not only affects, but also reflects corporate policies and value.

2.1 Discussion of the Assumptions

The model nests trading frictions faced by firm's shareholders and intermediaries into a dynamic corporate finance model with financial constraints. As such, the model is especially relevant for small firms, which suffer from severe financial constraints and whose stocks are relatively illiquid and traded at non-negligible bid-ask spreads.[9]

The modeling of secondary market transactions is flexible enough to be applicable to stocks traded on major exchanges as well as to stocks traded in over-the-counter markets. One key difference of this paper vis-à-vis models of the effects of liquidity demand/supply on asset valuations is the explicit focus on the policies of the issuing firm. Whereas previous contributions usually take the flow of dividends associated with a given stock as constant, the current paper endogenizes it. To keep the analysis simple, two assumptions are made. First, the trading costs borne by shocked (selling) investors are positive, whereas the costs borne by (buying) non-shocked investors are zero. This assumption is consistent with Brennan et al.

[9] Large firms have an easier access to several sources of financing (like corporate bonds or commercial paper), and their stocks are traded at tiny bid-ask spreads. Whereas large firms dwarf small firms capitalization-wise, the number of small and micro firms exceeds by far that of large firms, representing more than 80% of the number of listed firms.

(2012), who show that sell-order frictions are priced more strongly than buy-order ones.[10] Second, as in Budish et al. (2015) or Huang and Wang (2010), the model abstracts from asymmetric information about firm value. Indeed, recent evidence shows the importance of the non-information component of trading costs on asset prices (e.g., Chung and Huh, 2016). This paper builds on this strand and focuses on the implications for corporate outcomes.

Turning to the firm's financing, the paper assumes that the firm faces uncertainty in its ability to raise fresh funds as HMM, an issue that is especially severe for small firms, as also pointed out by the U.S. SEC Advisory Committee in Small and Emerging Companies. As illustrated by the survey evidence in Lins et al. (2010), financing uncertainty is one of the top reasons behind corporate cash stockpiling. Thus, the model realistically allows firm management to accumulate earnings in cash reserves to withstand operating losses. To assess the robustness to alternative modeling of the firm's financing frictions, Section 5.1 assumes that the firm faces issuance costs whenever raising new equity, as in DMRV and BCW. This extension confirms the qualitative and quantitative robustness of our results.

It is also worth noting that small/micro firms find it too costly (or unfeasible) to access bond financing. Rather, these firms usually access debt by borrowing from banks—e.g., by securing themselves credit line availability. While the baseline version of the paper abstracts from this source of financing, credit line availability is introduced in Section 5.3. The model predictions are shown to be robust to this extension.

Furthermore, whereas the baseline model assumes that the firm can expand the size of its operations through the exercise of a lumpy growth option, Section 5.2 considers continuous investment subject to adjustment costs, as BCW. This model extension allows to understand how trading frictions faced by firm investors (such as bid-ask spreads) affect the accumulation of capital by affecting the firm's investment/disinvestment rate, then strengthening and expanding the results of the baseline model.

Finally, whereas the baseline version of the model assumes that the mass of liquidity

[10] Brennan et al. (2012) show that the pricing of illiquidity emanates principally from the sell side. The underlying idea is that agents seldom face needs to buy stock urgently, but unexpected needs for cash may force them to suddenly sell stocks.

providers following the stock is constant and exogenous, Section 5.4 endogenizes it. In this framework, the probability with which the firm's shareholders trade with intermediaries is endogenous and varies over time. I show that the model predictions—in particular, the presence of a feedback effect between financial markets and corporate policies—are robust to this model extension.

3 The Effect of Bid-Ask Spreads on Corporate Policies

To disentangle the economic strengths at play, I start by analyzing the firm's optimal policies and value when shareholders face an exogenous bid-ask spread η. This assumption will be relaxed in Section 4.

3.1 Deriving Firm Value

Because liquidity shocks are independent across investors, a measure δdt of shareholders is shocked on each time interval. Shareholders seek to sell the stock as soon as hit by a liquidity shock. When trading with shocked shareholders, trading firms capture a fraction η of the surplus created, which means that the transaction price of the aggregate claim of shocked shareholders is $\delta\Phi(c) \equiv (1-\eta)\delta V(c)$. As long as $\delta\Phi(c) > (1-\chi)\delta V(c)$ (which is the case if $\eta < \chi$), liquidity-shocked shareholders are better off selling the stock (which entails the loss η) than keeping it (which entails the loss χ). The quantity

$$\delta[\Phi(c) - V(c)] = -\delta\eta V(c)$$

is then the associated (aggregate) expected loss to shocked shareholders at any time.

I next show that this loss affects corporate policies and value. Assume first that the firm does not have the growth option[11] and consider the optimal financial policies. As in previous cash management models, the benefit of holding cash decreases with cash reserves.

[11]Solving for firm value when there are no growth option is auxiliary to studying the optimal investment rule, as in HMM. The optimal investment rule is reported in Proposition 4.

Its (opportunity) cost is the wedge between the return required by the investors and the return on cash. I conjecture (and verify) that there is a target cash level, C_V, at which the cost and benefit of cash are equalized. Above C_V, it is optimal to pay excess cash out. Below C_V, shareholders retain earnings in cash reserves and search for financing. When operating losses cannot be covered by drawing funds from cash reserves or by raising fresh equity, the firm is forced into liquidation. Liquidation then occurs the first time that the cash reserves process hits zero:

$$\tau = \inf \{t \geq 0 : C_t \leq 0\} . \tag{5}$$

By Itô's lemma, firm value satisfies the following equation for any $c < C_V$:

$$\rho V(c) = (rc + \mu) V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda \sup_f [V(c+f) - f - V(c)] + \delta [\Phi(c) - V(c)] . \tag{6}$$

The left-hand side is the return required by the investors. The first two terms on the right-hand side represent the effect of cash retention and cash flow volatility on equity value. The third term represents the surplus from raising external financing, i.e., the probability-weighted surplus accruing to incumbent shareholders. In the appendix, $V(c) - c$ is shown to increase with c, so it is optimal to raise the cash buffer up to C_V whenever financing opportunities arise.[12] Thus, the optimal refinancing amount is $f(c, \Phi) = C_V(\Phi) - c$. The last term reflects the loss borne by liquidity-shocked investors when selling the firm's stocks. Substituting $\Phi(c)$ and $f(c, \Phi)$ into (6) gives the following ordinary differential equation (ODE):

$$(\rho + \delta\eta) V(c) = (rc + \mu) V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda [V(C_V) - V(c) - C_V + c] . \tag{7}$$

The left-hand side reveals that the bid-ask spread increases the return required by investors by $\delta\eta \geq 0$, consistent with Amihud and Mendelson (1986). This additional compensation leads to an increase in the opportunity cost of cash from $\rho - r$ to $\rho + \delta\eta - r$. Equation (7) is solved subject to the following boundary conditions. The firm is liquidated when cash is

[12] The marginal value of cash satisfies $V'(c) \geq 1$ (see Lemma 6 in Appendix A for a proof). This implies that the first derivative of $V(c) - c$ is non-negative.

exhausted and the firm cannot raise new funds. Thus,

$$V(0) = \ell \tag{8}$$

holds. Moreover, it is optimal to distribute all the cash exceeding C_V as payouts.[13] Firm value is thus linear for any $c \geq C_V$: $V(c) = V(C_V) + c - C_V$. Subtracting $V(c)$ from both sides of this equation, dividing by $c - C_V$, and taking the limit $c \to C_V$ gives

$$\lim_{c \uparrow C_V} V'(c) = 1. \tag{9}$$

That is, it is optimal to start paying out cash when the marginal value of one dollar inside the firm equals the value of a dollar paid out to shareholders. The target cash level that maximizes shareholder value is determined by the super-contact condition,

$$\lim_{c \uparrow C_V} V''(c) = 0. \tag{10}$$

Both the target cash level C_V and the issue size f depend on η. Below I further investigate corporate policies vis-à-vis an environment in which the bid-ask spread is zero.

3.2 Analyzing Corporate Policies

Cash Management and Payouts In the presence of financing frictions, the benefit of cash stems from guaranteeing financial flexibility to the firm. The firm's target cash level balances this benefit against the opportunity cost of holding cash. Notably, this model shows that the bid-ask spread associated with a firm's stock affects this tradeoff by impacting both the benefit *and* cost of cash. By increasing the return required by the investors, a positive bid-ask spread leads to: (i) an increase in the cost of equity (then increasing the benefit of holding cash), and (ii) an increase in the cost of cash, as it expands the wedge between the

[13]As shown in the following, it is never optimal to buy back the shares of shocked investors when $c < C_V$.

firm's cost of capital and the return on cash.[14] In this section, I analytically investigate the net effect of these strengths on the incentives to keep cash.

Consider first the extreme case in which financing frictions are so severe that the firm has no access to external financing (i.e., $\lambda = 0$). The bid-ask spread does not affect firm's refinancing decisions in this case (as the firm has no access to outside financing) and, thus, does not affect the benefit of cash directly. However, bid-ask spreads increase the cost of holding cash. As a result, the bid-ask spread leads to a decrease in the target cash level.

When the firm has access to external financing ($\lambda > 0$), the bid-ask spread has opposing effects on the incentive to keep cash. First, the larger opportunity cost of cash leads the firm to reduce its target cash level. Second, the larger return required by the investors reduces the surplus from outside financing accruing to existing shareholders. All else equal, this effect makes cash reserves more valuable and, thus, generates an incentive to increase the target cash level. The next proposition shows that the first effect dominates. I denote by C^* the target cash level when the bid-ask spread is zero.

Proposition 1 (Cash management) *On net, a positive bid-ask spread leads to a decrease in the target cash level, all else equal; i.e., the inequality $C_V \leq C^*$ holds. The greater the bid-ask spread η, the smaller the target level C_V.*

Proposition 1 illustrates that a positive bid-ask spread leads firms to decrease their optimal cash reserves (see Appendix A.1 for a proof). This prediction is consistent with Nyborg and Wang (2021), who find that stock liquidity (using the relative effective bid-ask spread as a measure, among others) increases a firm's propensity to hold cash.

Cash retention and payout decisions are closely related. I define the payout probability:

$$P^p(c, C_V) = E_c\left[e^{-\lambda \tau_d(C_V)}\right],$$

where $\tau_d(C_V)$ represents the first time that the cash reserves process, initially at c, reaches

[14] This model then delivers finite target cash levels even when r and ρ coincide. In previous dynamic cash management models, holding cash is not costly if r and ρ coincide and, thus, a financially constrained firm would pile infinite cash reserves if $r = \rho$ holds.

the payout threshold C_V. The bid-ask spread enters this probability through $\tau_d(C_V)$—by affecting the target cash level, the bid-ask spread affects the firm's payout frequency. The following proposition studies this probability compared to the case in which the bid-ask spread is zero (in which case the payout probability is denoted by $P^p(c, C^*)$).

Proposition 2 (Payout probability) *A positive bid-ask spread leads to an increase in the firm's payout probability, i.e.,* $P^p(c, C_V(\eta)) > P^p(c, C^*)$.

Proposition 2 implies that a firm pays out more dividends if its stock is traded at a larger bid-ask spread (see Appendix A.2). In so doing, the firm compensates shareholders for the frictions borne when trading the stock. This finding is in line with Banerjee et al. (2007), who contend that investors view stock market liquidity and dividends as substitutes. When a firm's bid-ask spread is small, investors can create dividends to themselves by cashing out their investment. When the bid-ask spread is large, investors require the firm to pay out more dividends.

One question then arises as to whether the firm should commit to repurchase the shares of shocked investors even when the cash reserves are below the target level, in order to reduce the return required by the investors and, thus, decrease its cost of capital. Appendix A.1.1 shows that this policy would be suboptimal, as cash would be paid out to shareholders when its marginal value is greater than one (meaning that cash is more valuable inside than outside the firm).[15] Instead of repurchasing shares for any level of cash reserves c, the optimal policy prescribes that the firm should "provide liquidity" to shareholders by reducing the target cash level with respect to the benchmark case with zero bid-ask spread (as from Proposition 1), meaning that the payout threshold is hit more often (as shown in Proposition 2).

Liquidation and Financing Because the bid-ask spread increases the cost of internal and external equity, the firm's financial resilience should also be affected. To investigate this important aspect, I study the firm's probability of liquidation and financing. I define the

[15] That is, payouts are suboptimal when cash reserves are below the target level C_V, as cash is more valuable inside the firm ($V'(c) > 1$) than if paid out for any $c < C_V$.

probability of liquidation while the firm is searching for external funds as

$$P^l(c, C_V(\eta)) = E_c\left[e^{-\lambda\tau(C_V)}\right]$$

and, complementarily, the probability of external financing is defined as $P^f(c, C_V(\eta)) = E_c\left[1 - e^{-\lambda\tau(C_V)}\right]$. The bid-ask spread enters these probabilities through $\tau(C_V)$, representing the first time that the cash process, reflected at C_V, is absorbed at zero. The following proposition studies these probabilities vis-à-vis the case in which the bid-ask spread is zero (see Appendix A.3).

Proposition 3 (Probability of liquidation and financing) *A positive bid-ask spread increases the firm's probability of liquidation, $P^l(c, C_V(\eta)) > P^l(c, C^*)$, and decreases the probability of external financing, $P^f(c, C_V(\eta)) < P^f(c, C^*)$.*

Taken together, these results suggest that bid-ask spreads exacerbate firms' financial constraints. First, bid-ask spreads increase the cost of internal financing, which reduces the size of the firm's optimal cash reserves, all else equal. Second, they reduce the firm's probability of external financing. As a result, bid-ask spreads increase the probability of firm's liquidation—and more so the wider the bid-ask spread is—consistent with the evidence in Brogaard et al. (2017).

Investment Decisions The analysis so far focuses on the case in which the firm has no growth opportunities (i.e., it conditions on a given drift μ_i). Indeed, the value of the firm with no growth opportunities serves to derive the zero-NPV cost—i.e., the maximum amount that the firm is willing to pay to increase the cash flow drift from μ to μ_+.[16] The next proposition illustrates how the bid-ask spread affects such zero-NPV cost (see Appendix A.4 for a proof).

Proposition 4 *A positive bid-ask spread reduces the maximum amount that the firm is will-*

[16]For instance, see Décamps and Villeneuve (2007) and HMM.

ing to pay to invest in the growth option. Specifically, the zero-NPV cost is given by

$$I_V = \frac{\mu_+ - \mu}{\rho + \delta\eta} - (C_{V+} - C_V)\left[1 - \frac{r}{\rho + \delta\eta}\right], \tag{11}$$

where C_{V+} denotes the target cash level after the growth option is exercised.

If the bid-ask spread was zero, the zero-NPV cost would be

$$I^* = \frac{\mu_+ - \mu}{\rho} - (C_+^* - C^*)\left(1 - \frac{r}{\rho}\right), \tag{12}$$

with C_+^* denoting the post-investment target cash level. Comparing (11) and (12) reveals that a positive bid-ask spread leads to a decrease in the investment reservation price (i.e., $I_V < I^*$)—that is, it reduces the maximum amount that the firm is willing to pay to exercise the growth option. If the investment cost lies in $[I_V, I^*]$, the growth option has negative NPV if the bid-ask spread is positive ($\eta > 0$), whereas it has positive NPV if the bid-ask spread is zero. Moreover, the gap between the zero-NPV costs I_V and I^* increases with η. That is, the underinvestment problem worsens if the bid-ask spread widens, a result that is empirically consistent with Campello et al. (2014) and Amihud and Levi (2019). Section 5.2 confirms and extends this prediction to the case in which the firm optimally adjusts its investment rate continuously, as in the neoclassical framework of Bolton et al. (2011).

Quantitative Analysis I now provide a quantitative assessment of the model results so far, i.e., when assuming that the bid-ask spread is exogenous. Table I reports the baseline parameterization. The risk-free rate ρ is set to 2%, and the return on cash is set to 1%. The resulting opportunity cost of cash is equal to 1%, as in BCW and DMRV. Because small firms tend to have lower profitability in the cross section (see, e.g., Fama and French, 2008), the cash flow drift $\mu = 0.05$ is set to be lower than the value used by DMRV and consistent with the bottom range of values in Whited and Wu (2006). Upon exercising the growth option, the cash flow drift is assumed to be 20% bigger (i.e., $\mu_+ = 0.06$). I set $\sigma = 0.12$, which is consistent with Graham et al. (2015) and is higher than the value set by DMRV, as small firms have more volatile cash flows. Furthermore, I base liquidation costs on the estimates of Glover (2016) and set $\phi = 0.55$. The parameter λ is set to 0.75, which is consistent with the frequency of equity issues by small firms reported by Fama and French (2005). The intensity of the liquidity shock is set to $\delta = 0.7$, as in He and Milbradt (2014). The magnitude of the bid-ask spread is varied extensively throughout the analysis.[17]

Table II illustrates the impact of bid-ask spreads of various magnitude on the target cash level, the probability of external financing, the probability of liquidation, the probability of payout, the investment reservation price (i.e., the zero-NPV cost), and firm value. For a bid-ask spread equal to 60 basis points, the target cash level decreases by more than 13% with respect to the case in which the bid-ask spread is zero. Because the bid-ask spread engenders a wedge between the return required by the investors and the return on cash, the firm pays out cash to investors more often. In fact, the table shows that the probability of payout increases on average by about 3.1% when the bid-ask is equal to 60 basis points.[18] Moreover, the maximum investment threshold decreases by almost 18%, engendering a notable underinvestment problem.

Because both external and internal financing are more expensive if the bid-ask spread is positive, the probability of liquidation increases by 2%, on average. The increase in the probability of liquidation is greater as cash reserves approach depletion. Table III illustrates the impact of bid-ask spreads on the probability of liquidation, at different cash levels. When the firm stands at $C_V/4$, the probability of liquidation is equal to 14.1% if the bid-ask spread is zero, and is equal to 18.3% if the bid-ask spread is equal to 60 basis points. It is also worth noting that, for a given cumulative shock, liquidation becomes relatively more likely if the bid-ask spread is larger. If the bid-ask spread is zero, a series of shocks reducing the cash

[17] Chung and Zhang (2014) report the median bid-ask spread (calculated using TAQ data) for firms sorted by quintiles of market capitalization over the period 1993-2009. They report that the median bid-ask spread of smaller quintile firms is 0.0195 for NYSE/AMEX stocks and 0.0501 for NASDAQ stocks. They also note that the bid-ask spread has decreased over time (see also Hasbrouck, 2009): The median bid-ask spread for (all capitalization) NYSE/AMEX stocks went from 0.0094 in 1993 to 0.0034 in 2009, and from 0.0346 in 1993 to 0.0067 in 2009. In the model parameterization, I take a conservative approach and take a relatively low value for the bid-ask spread. In so doing, I show that even small bid-ask spreads can bear substantial impact on corporate policies and value.

[18] To calculate these probabilities, I follow HMM and calculate the average for a cross-section of firms with cash reserves uniformly distributed between 0 and C_V.

buffer from $C^*/2$ to $C^*/4$ increases the probability of liquidation from 1.97% to 14.1%. If the bid-ask spread is equal to 60 basis points, the reduction from $C_V/2$ to $C_V/4$ is caused by a cumulative loss about 13% smaller (than in the case in which the bid-ask spread is zero), but it increases the probability of liquidation from 3.35% to 18.3%.

The analysis then advances the following testable predictions. First, firms whose stocks trade at a larger bid-ask spread are more financially constrained, as they are less likely to tap the equity market and keep less precautionary cash. Second, the firm is less resilient to negative operating shocks, and the probability of liquidation increases more steeply after a given cumulative shock. Third, the firm underinvests, as the additional return required by the investors erodes the profitability of investment opportunities and reduces the firm's reservation price. Overall, firm value decreases considerably. For instance, a bid-ask spread equal to 60 basis points leads to an 18% reduction in firm value. Table II shows that such a loss is sizable even for smaller bid-ask spreads.

4 Endogenizing the Bid-Ask Spread

Having analyzed how bid-ask spreads affect corporate financial policies, liquidation risk, investment, and firm value, I now derive the equilibrium bid-ask spread by assuming participation frictions faced by liquidity providers. That is, in this section, η is endogenous—thus, not only it affects, but also reflects firm value. (Alternatively, Section 5.4 endogenizes the mass of liquidity providers actively following the stock, showing the robustness of the model predictions to this alternative specification.)

4.1 Endogenous Liquidity Provision and Corporate Policies

Trading firms are liquidity providers that are active on both the bid and ask sides of transactions. (In the following, trading firms and liquidity providers will be used interchangeably.) As in Section 3, the ask price is equal to the fundamental value of the firm, $V(c)$, as trading firms sell stocks to non-liquidity-shocked investors. On the bid side, trading firms buy stocks from liquidity-shocked investors, so they can extract rents from this side of transactions. At any time, the equilibrium bid-ask spread is determined by the zero-profit condition, which is given by:

$$-\delta\,(1-\eta)\;V(c;\eta,C_V)+\delta V(c;\eta,C_V)(1-\kappa)=\gamma \quad s.t. \quad \eta\leq\chi. \tag{13}$$

The left-hand side represents the expected net gain from intermediating. Namely, the first term is the price at which trading firms buy the stock from liquidity-shocked shareholders, whereas the second term is the price at which trading firms sell the stock to non-shocked investors net of their funding cost. The right-hand side of this equation is the participation cost borne by trading firms to maintain constant market presence. The equilibrium η cannot exceed χ, otherwise shocked shareholders would be better off holding the stock instead of selling it to trading firms. Notably, the endogenous bid-ask spread set by trading firms affects *and* depends on firm value.

Using standard arguments, firm value satisfies the following HJB equation:

$$\begin{aligned}\rho V(c;\eta) &= (rc+\mu)\,V'(c;\eta)+\frac{\sigma^2}{2}V''(c;\eta)+\lambda\sup_f[V(c+f;\eta)-V(c;\eta)-f]\\ &\quad+\delta\left[(1-\min[\eta(c);\chi])V(c,\eta)-V(c,\eta)\right].\end{aligned} \tag{14}$$

This equation admits an interpretation similar to equation (6). As in the case with constant (exogenous) η, it is optimal to raise funds up to the target level whenever financing opportunities arise, i.e., $f=C_V(\eta)-c$. Also, similar to equation (6), the last term on the right-hand side is the loss borne by liquidity-shocked shareholders. Differently, $\eta(c)$ is now endogenous. Through equation (13), the equilibrium bid-ask spread that makes the zero-profit condition binding satisfies:

$$\eta(c)=\min\left[\chi,\frac{\gamma}{\delta V(c)}+\kappa\right]. \tag{15}$$

This expression captures the real-world observation that bid-ask spreads are wider for smaller capitalization firms, as shown by Hasbrouck (2009) and Chung and Zhang (2014) among others. Notably, cash flow volatility, firm profitability, and the severity of financing frictions affect the bid-ask spread through their impact on firm value, $V(c)$. Equation (15) also illustrates that larger participation or financing frictions faced by liquidity providers (γ and κ) lead to greater bid-ask spread—as shown in the following, it does so both directly and through their effect on firm value.

Whenever $\eta(c) < \chi$, equation (14) can be re-written as follows:

$$(\rho + \delta\kappa)V(c) = (rc + \mu)\, V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda \left[V(C_V) - C_V + c - V(c) \right] - \gamma \qquad (16)$$

This equation illustrates that the frictions borne by liquidity providers are passed on to the liquidity-shocked shareholders. As a result, frictions affecting the provision of liquidity end up affecting firm value. If $\eta(c)$ is never binding at χ (i.e., $\eta(c) < \chi$ for any $c < C_V$), equation (16) is solved subject to conditions similar to those in Section 3, i.e. $V(0) = \ell$ and $\lim_{c \uparrow C_V} V'(c) = 1$ at the liquidation threshold and at the target cash level, respectively. Again, the target cash level is identified by the super-contact condition, $\lim_{c \uparrow C_V} V''(c) = 0$.

Consider now the case in which the bid-ask spread binds at χ for some $c < C_V$. Equation (15) suggests that η can become binding at χ if the participation fees or funding costs faced by trading firms are sufficiently large, and firm value is sufficiently low. Namely, if firm value falls below the critical value $\underline{V} = \frac{\gamma}{\delta(\chi - \kappa)}$, then $\eta(c) = \chi$. Whenever $\eta(c) = \chi$, investors are indifferent between selling the stock or keeping it. This is consistent with the real-world observation that bid-ask spreads do not increase unboundedly but, especially for small firms, trading volume wanes if bid-ask spreads grow too large. In these cases, firm value satisfies:

$$\rho V(c) = (rc + \mu)\, V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda \left[V(C_V) - C_V + c - V(c) \right] - \delta\chi V(c). \qquad (17)$$

As shown in Appendix B, there is at most one threshold $\underline{C} \in [0, C_V]$ such that $V(\underline{C}) = \underline{V}$. For $c \in (\underline{C}, C_V]$, the bid-ask spread $\eta(c)$ is smaller than χ, so shareholders strictly prefer selling the stock than keeping it (which entails the cost χ) and firm value satisfies equation (16). Conversely, if $c \in [0, \underline{C})$, liquidity shocked shareholders are indifferent between keeping the stock (then bearing the opportunity cost χ) or bearing the bid-ask spread $\eta = \chi$. Whenever $\eta(c) = \chi$ in a right interval of $c = 0$, continuity and smoothness determine how the two ODEs, equations (17) and (16), are pasted together (see Appendix B for analytical details).

4.2 Implications

Feedback Effects Section 3 shows that bid-ask spreads lead shareholders to require an additional compensation to invest in the firm, in line with Amihud and Mendelson (1986) and Acharya and Pedersen (2005). This additional compensation constraints corporate policies—leading to tighter financial constraints and a larger probability of liquidation—then decreasing firm value. Building on this analysis, this section shows that when liquidity provision is endogenous, bid-ask spreads not only affect but also are affected by firm value, as follows.

When liquidity provision is endogenous, the drop in firm value driven by the bid-ask spread (analyzed in Section 3) is factored into the liquidity providers' zero-profit condition (equation (13)). Specifically, it leads liquidity providers to extract more rents from shocked investors as a proportion of the value of their claim to cover participation and funding costs. The proportional bid-ask spread borne by the firm shareholders rises and exacerbates the detrimental effects of the bid-ask spread illustrated in Section 3. Specifically, the firm's financial constraints tighten further (the cost of internal and external financing increase, and the target cash level is adjusted downwards), and the probability of liquidation rises. The ensuing further decrease in firm value feeds back again into the participation constraints of intermediaries. This feedback loop then leads to a larger bid-ask spread, more severe financial constraints, and a lower firm value compared to the case in which liquidity provision is exogenous.

Reinforcing Constraints By affecting the equilibrium bid-ask spread, participation frictions faced by liquidity providers impact corporate policies. Table IV shows that larger γ or κ lead to a decrease in the target cash level, an increase in the probability of payout, and a rise in the probability of liquidation. In other words, in the presence of the feedback effect described above, trading frictions exacerbate the firm's financial constraints by increasing the firm's cost of internal and external financing. In turn, more severe financial constraints reduce firm value and lead to a larger bid-ask spread—that is, trading frictions and financial constraints reinforce each other. Table IV also shows that frictions borne by liquidity providers have a substantial impact on the firm's investment decisions, by leading to a sharp reduction in the maximum price that the firm is willing to pay to increase the cash flow drift from μ to μ_+.[19] Overall, frictions faced by liquidity providers have a substantial, detrimental effect on firm value, as quantified in the last column of Table IV.

Figure 1 further investigates the effects of γ or κ on the firm's probability of liquidation, external financing, and payout. It illustrates that the probability of liquidation increases with γ and κ—also, it is greater than in the case in which the bid-ask spreads is zero. In addition, by leading to an increase in the bid-ask spread and, thus, in the return required by the investors, frictions borne by liquidity providers lead to a decrease in the firm's probability of external financing, which is lower than in the benchmark case in which the bid-ask spread is zero. Finally, by increasing the opportunity cost of cash, frictions borne by liquidity providers lead to an increase in the firm's payout probability. This analysis illustrates that frictions borne by liquidity providers are eventually passed on to investors and substantially affect corporate policies and value, consistent with Goldberg (2020).

Amplification of Shocks Figure 2 and 3 show the endogenous bid-ask spread and firm value for different values of the participation fee γ and funding cost κ borne by liquidity providers. The top panels show that bid-ask spreads are wider when participation fees or funding costs faced by liquidity providers are greater. This result is consistent with Comerton-Forde et al. (2010), Hameed et al. (2010), and Cotelioglu et al. (2021), who show that market makers' financial constraints affect the liquidity of their traded stocks.

Notably, the model shows that liquidity providers' participation frictions impact the firm bid-ask spread through a direct and an indirect channel. Consider the effect of an increase

[19] The expression for the maximum zero-NPV cost is provided in Appendix B and is analogous to that reported in Proposition 4.

in the frictions γ or κ. Such an increase implies that liquidity providers extract a relatively larger share of the value of the claim of shocked shareholders to remain active in the market of the stock. The bid-ask spread then widens (see equation (13)). This is the direct effect. Yet, this wider bid-ask spread leads to a reduction in firm value (as shown in Section 3), which prompts liquidity providers to charge an even larger bid-ask spread. This is the indirect effect. Quantitatively, an increase in γ from 0.7% to 0.8% would lead to a 9.1% increase in the bid-ask spread if firm value was independent of γ, i.e., simply accounting for the direct effect described above.[20] However, when accounting for its impact on firm value (i.e., the indirect effect), the bid-ask spread is 10.8% bigger as a result of the increase in γ. Similarly, an increase in κ from 0.3% to 0.4% would lead to a 12.2% increase in the bid-ask spread when simply accounting for the direct effect. When accounting for its indirect effect through firm value, the bid-ask spread increases by 14.4%.

That is, the model shows that shocks that affect the participation of liquidity providers (e.g., making market presence more costly) become more severe when reflected into firm value, then illustrating a novel mechanism through which shocks originating in financial markets propagate to corporations, eventually being amplified. In particular, this amplification mechanism contributes to explain why small firms' bid-ask spreads increase the most (i.e., much more than those of large firms) when liquidity providers' funding constraints tighten, as documented by Anand et al. (2013) and Aragon and Strahan (2012). Indeed, whereas the model can apply to any firm whose stock's bid-ask spread is non-negligible, it is highly relevant for small firms, that indeed face the largest bid-ask spreads in the cross-section and have uncertain access to outside financing.

Figure 2 and 3 also illustrate that the bid-ask spread reacts asymmetrically to negative or positive cash flow shocks leading, respectively, to a decrease or an increase in the firm's cash reserves and firm value. Namely, the increase in the bid-ask spread following a negative shock is greater in absolute magnitude than the decrease following a positive shock of the same size. This result is consistent with Hameed et al. (2010), showing that liquidity responds

[20] For the sake of fixing ideas, firm value is calculated at the midpoint of cash reserves, $c = C_V/2$, in these numerical examples.

asymmetrically to shocks to asset values, deteriorating more sharply after negative ones.

4.3 Application: Firm-funded Designated Market Makers (DMM)

Competitive market forces have been shown to potentially lead to inefficient liquidity provision and suboptimal market outcomes (or market failures), see for instance Bessembinder et al. (2015).[21] This issue has raised the attention of policy-makers. For example, the recommendations of the SEC Advisory Committee on Small and Emerging Companies are based on the idea that competitive market forces may break down when it comes to small or micro capitalization firms. Partially addressing this issue, stock markets in several European countries have contemplated a contract, whereby listed firms pay a DMM to maintain the bid-ask spread below a given (contractual) threshold and enhance the liquidity of the firm's stock. In this section, the model is extended to study the desirability of this policy provision from the perspective of financially constrained firms.

Consider the contract between a listed firm and a DMM. Similar to Bessembinder et al. (2015), I assume that the DMM is required to keep the bid-ask spread within a specific width, in exchange for a rent that is paid by the firm. Consistently, I assume that the firm pays the DMM a periodic payment Γ to keep the bid-ask below a given level $\bar{\eta}$, which is assumed to be smaller than χ, to consider the relevant case. That is, $\eta(c) \leq \bar{\eta} < \chi$ for any c. In this setup, the equilibrium bid-ask spread solves:

$$-\delta\,(1-\eta)\;V(c;\eta,C_V(\eta)) + \delta V(c;\eta,C_V(\eta))(1-\kappa) = \gamma - \Gamma \quad s.t. \quad \eta \leq \bar{\eta} < \chi. \tag{18}$$

The above condition differs from (13) in two main aspects. First, the participation fee borne by liquidity providers is partly financed by the firm (i.e, the right-hand side of this equation is just $\gamma - \Gamma$). Second, the contractual payment Γ guarantees that the equilibrium η cannot exceed $\bar{\eta}$ (see Appendix B.1). Intuitively, if $\bar{\eta}$ is smaller (meaning that DMM are required to keep the bid-ask spread more narrow), then the periodic fee Γ is larger (i.e., it is more costly

[21] Bessembinder et al. (2015) show that competitive liquidity provision in secondary markets is associated with reduced welfare and a discounted secondary market price that can potentially dissuade IPOs.

to enter the DMM contract from the firm perspective).

By condition (18), firm value is shown to satisfy the following equation:

$$(\rho + \delta\kappa)V(c) = (rc + \mu - \Gamma)\,V'(c) + \frac{\sigma^2}{2}V''(c) + \lambda\,[V(C_V) - C_V + c - V(c)] - (\gamma - \Gamma). \quad (19)$$

This equation shows that the periodic rent paid by the firm to the DMM reduces the extent to which participation costs are passed on to shareholders (the last term on the right-hand side is $\gamma - \Gamma$, whereas it is equal to γ if the firm does not enter the DMM contract). However, the first term on the right-hand side of equation (19) illustrates that the payment to the DMM, Γ, drains the firm's periodic cash flow. The following result is shown in Appendix B.1.

Proposition 5 *Entering the DMM contract does not enhance the value of financially constrained firms, on net.*

Firms face a tradeoff when considering the DMM contract. On the positive side, entering the contract reduces the bid-ask spread borne by firm shareholders, which leads to a decrease in the cost of internal and external financing (as illustrated so far). On the negative side, entering the DMM contract drains the firm's cash flows, which in turn makes the firm more financially constrained. Because the marginal value of cash is greater than one for financially constrained firms (see Lemma 6 in Appendix A), cash is more valuable inside the firm than if used to fund the DMM. As a result, the negative effect dominates the positive effect.

Figure 4 compares the equilibrium bid-ask spread and firm value when the firm does and does not enter the DMM contract.[22] The top panel shows that when the firm enters the contract, the equilibrium bid-ask spread decreases, which is beneficial to the firm as it decreases its cost of capital. Yet, the firm has to give up some of its revenues to fund the DMM. On net, the bottom panel of Figure 4 shows that firm value is almost unchanged, being slightly smaller if the firm enters the contract, consistent with Proposition 5.

[22]When the firm does not enter the DMM contract, the bid-ask spread is solved as in Section 4.1.

5 Robustness to Alternative Setups

5.1 Modeling Financing Frictions as Issuance Costs

The baseline model features financing frictions as capital supply uncertainty, consistent with the difficulties faced by small firms in raising external funds. This section alternatively models financing frictions as issuance costs, as DMRV or BCW. In this setup, the firm chooses the timing of equity issuances (rather than waiting for stochastic financing opportunities).[23]

Specifically, I assume that external financing entails proportional and fixed costs, denoted by ϵ and ω, respectively. These costs prompt the firm to keep precautionary cash reserves with a target level C_V. When it is optimal for the firm to save earning in the cash reserves (i.e., for any $c < C_V$), firm value satisfies the following equation:

$$\rho V(c) = (rc + \mu) V'(c) + \frac{\sigma^2}{2} V''(c) + \delta [\Phi(c) - V(c)], \tag{20}$$

which differs from equation (6) as there is no jump in firm value due to stochastic financing opportunities. Rather, the firm sets financing decisions to minimize issuance costs. To economize on the fixed cost, the firm raises funds in a lumpy fashion when cash reserves are depleted. Denote the optimal issue size by C_*. The following condition then holds:

$$V(0) = V(C_*) - (1 + \epsilon)C_* - \omega$$

which implies that firm value at $c = 0$ (the left-hand side of this equation) equals the firm's continuation value net of issuance cost (the right-hand side). Notably, it is optimal for the firm to raise external financing if the following inequality $V(C_*) - (1 + \epsilon)C_* - \omega > \ell$ holds, which guarantees that the firm's continuation value (the left-hand side) is larger than the liquidation value (the right-hand side). The optimal issue size C_* satisfies $V'(C_*) = 1 + \epsilon$, which warrants that the marginal benefit (the left-hand side of this equation) and cost of external financing (the right-hand side) are equalized at C_*. Lastly, Equation (20) is subject

[23]For simplicity, in this section I focus on the case in which the bid-ask spread is exogenous.

to boundary conditions at the target cash level that are similar to (9) and (10).

Table V shows the effect of bid-ask spreads on the target cash level, the optimal issuance size, and the zero-NPV investment cost. I use the baseline parameters in Table I and, in addition, consider two sets of values for the issuance costs. In the top panel, I set $\epsilon = 0.06$ and $\omega = 0.01$ as in BCW, whereas in the bottom panel I set $\epsilon = 0.10$ and $\omega = 0.03$, which account for the heterogeneity in financing costs documented by Hennessy and Whited (2007). Table V confirms that the target cash level decreases with the bid-ask spread. For a 60-basis-points bid-ask spread, the target cash level decreases by about 9% with respect to the case with zero bid-ask spread. Furthermore, this setup allows to investigate how bid-ask spreads affect the optimal size of equity issues (C_*). Table V illustrates that the size of equity issuances decreases with the bid-ask spread. For a 60-basis-points bid-ask spread, the optimal size of equity issuances is about 4% lower compared to the benchmark case in which the bid-ask spread is zero. Table V also confirms that the maximum investment cost decreases with the magnitude of the bid-ask spread. For a bid-ask spread equal to 60 basis points, the zero-NPV cost decreases by about 17% compared to the case in which the bid-ask spread is zero. This analysis then confirms that the impact of bid-ask spreads holds irrespective of the way financing frictions are modeled—i.e., being costs or uncertainty in raising new funds.

5.2 Continuous Investment and Capital Accumulation

I now relax the assumption that the firm has just one lumpy investment opportunity. Instead, following BCW, I assume that the firm can continuously adjust its capital stock, denoted by K_t. Specifically, I assume that the firm's capital stock evolves as follows:

$$dK_t = (i - d)K_t dt, \tag{21}$$

where i denotes the firm's endogenous investment rate, and d represents the depreciation rate of capital. Following BCW, the price of capital is normalized to one. Moreover, investment in capital entails a quadratic adjustment cost equal to $G(i, K) = g(i)K = \frac{\theta}{2} i^2 K$. As a result, the dynamics of the firm's operating profit satisfy

$$dX_t = K_t \left[(\mu - i - g(i))dt + \sigma dZ_t \right], \tag{22}$$

i.e., operating profits are proportional to the capital stock. The parameters $\mu > 0$ and $\sigma > 0$ are constants, and Z_t is a standard Brownian motion, similar to equation (1).

In this setup, firm value $V(C, K)$ is a function of cash reserves and of the capital stock. Standard arguments give the HJB equation reported in Appendix C.1. Exploiting homogeneity, the firm's optimization problem reduces to one state variable, being cash reserves normalized by capital $w = C/K$. Consistently, $v(w)$ is defined as firm value scaled by capital K_t. Calculations reported in Appendix C.1 show that the firm's optimal investment rate is:

$$i(w) = \frac{1}{\theta} \left(\frac{v(w)}{v'(w)} - 1 - w \right). \tag{23}$$

Figure 5 investigates how bid-ask spreads affect firm value (scaled by capital) and the optimal investment rate. I use the same parameterization as BCW,[24] to which I add secondary market frictions and financing uncertainty as in Table I. The left panel shows that scaled firm value decreases with the magnitude of the bid-ask spread, consistent with the results in Section 3.2. The right panel illustrates the optimal investment rate as a function of scaled cash reserves. The figure shows that the firm may have incentives to disinvest when its cash reserves are low, in which case financial constraints are more severe, similar to BCW. In the cash region characterized by positive investment, the optimal $i(w)$ is smaller if the bid-ask spread is wider. At the target cash level, the optimal investment rate is equal to 10.7% when the bid-ask spread is zero, and equal to 6.37% and 3.61% if the bid-ask spread is, respectively, 60 and 120 basis points. The figure also confirms that the target cash level is smaller if the bid-ask spread is wider. Importantly, this analysis demonstrates that bid-ask spreads not only affect the firm's incentives to keep cash as a store of value, but also the incentives to accumulate capital, then giving rise to a substantial underinvestment problem.

[24]Details are reported in Appendix C.1.

5.3 Debt Financing (Bank Credit Lines)

Small firms typically do not have access to the corporate bond or commercial paper markets, and are more likely to tap debt financing by drawing funds from bank lines of credit. A credit line is a source of funding that the firm can access at any time up to a pre-established limit L. Whenever the credit limit is finite ($L < \infty$), the firm has a positive demand for cash. As shown by BCW, this is true for exogenous or endogenous (value-maximizing) L.[25] In this section, I assess the model results in the presence of this additional source of financing.

I follow BCW and assume that the firm pays a constant spread, β, over the risk-free rate on the amount of credit used. Because of this cost, it is optimal for the firm to tap the credit line only when cash reserves are exhausted. The firm then uses cash as the marginal source of financing if $c \in [0, C_V(L)]$ (the cash region), where $C_V(L)$ denotes the target cash level in this environment. Conversely, the firm draws funds from the credit line when $c \in [-L, 0]$ (the credit line region). Firm value satisfies (7) in the cash region, whereas it satisfies

$$(\rho + \delta\eta)\, V(c) = [(\rho + \beta)c + \mu]\, V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda\, [V(C_V) - V(c) - C_V + c] \tag{24}$$

in the credit-line region. On top of the smooth-pasting and super-contact conditions at $C_V(L)$ similar to (9) and (10), the system of ODEs (7)–(24) is solved subject to the following boundary conditions. The first condition, $V(-L) = \max\,[\ell - L, 0]$, means that if $\ell \geq L$, the credit line is fully secured. Moreover, the conditions $\lim_{c\uparrow 0} V(0) = \lim_{c\downarrow 0} V(0)$ and $\lim_{c\uparrow 0} V'(0) = \lim_{c\downarrow 0} V'(0)$ guarantee continuity and smoothness at the point where the cash and the credit line regions are pasted.

Figure 6 studies the impact of bid-ask spreads when allowing for credit line availability. I use the parametrization in Table I and set $L = 0.08$ and $\beta = 1.5\%$ (see Sufi, 2009). The figure shows that the effects of bid-ask spreads are similar irrespective of the firm's access to bank credit. Access to credit relaxes the precautionary demand for cash, leading to a lower

[25] Firms often face credit supply frictions that prevent them from taking the value-maximizing limit L. Endogenizing L is an interesting extension to understand the relation between stock liquidity and the firm's willingness to access bank credit, and I leave it for future research.

target cash level. Still, the target cash level continues to be decreasing with the bid-ask spread. The probability of liquidation and payout as well as investment decisions are almost unchanged when allowing for firm's access to credit lines.

5.4 Endogenizing the Mass of Liquidity Providers

Section 4 endogenizes the liquidity of the firm's stock by assuming that liquidity providers are a fixed mass and set the bid-ask spread charged to the firm's shareholders competitively. In this section, I alternatively endogenize the mass of active liquidity providers in the market of the stock when they take the bid-ask spread η as given. The feedback effect between financial markets and firm policies arises in this setup too.

I assume that liquidity providers are an infinite and atomless mass but, because of participation costs, only a finite measure is active. The mass of liquidity providers in the market of the stock, denoted by θ_t, determines the probability with which shocked shareholders trade with these liquidity providers rather than keeping the asset. I define this probability as

$$\pi_t \equiv \frac{\theta_t}{\alpha + \theta_t}, \tag{25}$$

where $\alpha > 0$ captures inefficiencies that, all else equal, reduce π_t (e.g., rigidities in trading protocols, technological deficiencies, among others). This specification captures the notion of competition by order flow, as the probability with which a trading firm contacts investors, $\frac{\pi_t}{\theta_t}$, decreases with θ_t, similar to Lagos and Rocheteau (2007).

Competition in the market for liquidity provision implies that the mass of trading firms following the stock is determined by the zero-profit condition (as Lagos and Rocheteau, 2007):

$$\frac{\pi(\theta)}{\theta} \delta V(c; \theta, C_V)(\eta - \kappa) = \gamma \tag{26}$$

The left-hand side of this equation is the expected rent to an active liquidity provider, which is equal to the probability-weighted gain from trading η (which is now taken as given) net of the cost of funding κ.[26] The right-hand side is the cost of being active in the market of the stock. Through (26), the equilibrium mass of liquidity providers and the ensuing probability of intermediated trading for shocked investors are:

$$\theta(c) = \left(\frac{\delta(\eta-\kappa)}{\gamma}V(c) - \alpha\right)^+, \qquad \pi(c) = \left(1 - \frac{\alpha\gamma}{\delta(\eta-\kappa)V(c)}\right)^+. \tag{27}$$

The mass of active liquidity providers must be non-negative, which implies that trading firms participate in the market for the stock if firm value is larger than a critical value defined by $\underline{V}_\theta = \frac{\alpha\gamma}{\delta(\eta-\kappa)}$. This critical value increases with the frictions faced by trading firms (γ, κ, and α). Notably, as in the baseline model analyzed in Section 4.1, there is a unique threshold $\underline{C}_\theta$ such that $V(\underline{C}_\theta) = \underline{V}_\theta$.[27] If $c > \underline{C}_\theta$, the measure of active liquidity providers $\theta(c)$ (and the probability $\pi(c)$) is positive, non-decreasing, and concave in c, and the expected loss associated with shareholders' liquidity shocks is $\left[\pi(c)\eta + \big(1-\pi(c)\big)\chi\right]\delta V(c)dt$. In this expression, the first term represents the fee associated with trading with a liquidity provider times its probability, whereas the second term represents the loss associated with keeping the stock. Conversely, for any $c \in [0, \underline{C}_\theta]$, the probability of intermediated trading is zero, so the cost borne by liquidity shocked shareholders is equal to χ. Appendix C.2 reports the analytical details of solving for firm value.

In this alternative setup, the feedback effect illustrated in Section 4 arises too. In fact, the illiquidity-driven drop in firm value analyzed in Section 3 shrinks the expected rents to liquidity providers and affects their participation (see equation (26)). As the expected flow of rents to liquidity providers decreases, some of them stay away from the market of the stock. Liquidity provision wanes, and the expected loss faced by shocked shareholders becomes more severe. That is, as in the baseline in Section 4, the feedback effect makes trading more costly for shareholders in expectation. Differently, it does so through the lower participation of liquidity providers. Again, the feedback effect implies scarcer liquidity in the market of the stock, as well as lower firm value and more severe financial constraints

[26] To consider the relevant case, throughout we assume that $\eta > \kappa$.

[27] In this alternative setup, the critical value $\underline{V}_\theta$ is analogous to $\underline{V}$, and the critical threshold $\underline{C}_\theta$ is analogous to $\underline{C}$ defined in Section 4.1.

compared to the case in which liquidity is exogenous.

6 Concluding Remarks

This paper develops a model that studies the intertwined relation between trading and financing frictions and analyzes their real effects. The model shows that bid-ask spreads increase the firms' cost of equity *and* the opportunity cost of cash. As such, they make firms more financially constrained—as they lead firms to keep smaller cash reserves and to be less likely to raise external financing—and more exposed to forced liquidations. These firms also face a severe underinvestment problem, as the additional return required by the investors erodes the profitability of investment opportunities. Overall, firm value decreases. The model also shows that when liquidity providers face participation frictions and set the bid-ask spread competitively, this drop in firm value feeds back into the magnitude of the bid-ask spread. This mechanism implies that frictions faced by liquidity providers are passed on to investors and, through this channel, have an important impact on the policies, values, and survival rates of small firms. The model illustrates a channel through which frictions in the financial sector propagate to the corporate sector, eventually being amplified—in particular, the model shows that trading and financing frictions reinforce each other. More generally, the model suggests that the architecture of secondary market transactions has a prime effect on corporate decisions, especially for firms that face severe financing frictions.

Appendices

A Proof of the Results in Section 3

Throughout the Appendix, I define the quantity $\Phi \equiv \delta\eta$ to ease the notation. I start by proving that $V(c)$ is increasing and concave for any $c < C_V$.

Lemma 6 *$V'(c) > 1$ and $V''(c) < 0$ for any $c \in [0, C_V)$.*

Proof. Simply differentiating the following equation (which is equivalent to equation (7))

$$(\rho + \lambda + \Phi)\ V(c) = V'(c)\,(rc + \mu) + \frac{\sigma^2}{2}V''(c) + \lambda\Big[V(C_V) - C_V + c\Big], \tag{28}$$

one gets

$$(\rho + \lambda + \Phi - r)\,V'(c) = V''(c)\,(rc + \mu) + \frac{\sigma^2}{2}V'''(c) + \lambda.$$

By the conditions $V'(C_V) = 1$ and $V''(C_V) = 0$, it follows that $V'''(C_V) = \frac{2}{\sigma^2}\,(\rho + \Phi - r) > 0$ as $r < \rho$, meaning that there exists a left neighborhood of C_V such that for any $c \in (C_V - \epsilon, C_V)$, with $\epsilon > 0$, the inequalities $V'(c) > 1$ and $V''(c) < 0$ hold. Toward a contradiction, I assume that $V'(c) < 1$ for some $c \in [0, C_V - \epsilon]$. Then there exists a point $C_c \in [0, C_V - \epsilon]$ such that $V'(C_c) = 1$ and $V'(c) > 1$ over (C_c, C_V), so

$$V(C_V) - V(c) > C_V - c \tag{29}$$

for any $c \in (C_c, C_V)$. For any $c \in (C_c, C_V)$ it must be also that

$$V''(c) = \frac{2}{\sigma^2}\left\{(\rho + \lambda + \Phi)\,V(c) - [rc + \mu]V'(c) - \lambda(V(C_V) + c - C_V)\right\}$$

Using (29), jointly with $V(C_V) = \frac{rC_V + \mu}{\rho + \Phi}$, it follows that

$$V''(c) < \frac{2}{\sigma^2}\left\{(\rho + \Phi)(V(C_V) + c - C_V) - rc - \mu\right\} = \frac{2}{\sigma^2}(c - C_V)(\rho + \Phi - r) < 0.$$

This means that $V'(c)$ is decreasing for any $c \in (C_c, C_V)$, which contradicts $V'(C_c) = V'(C_V) = 1$. It follows that C_c cannot exists. So, $V'(c) > 1$ and $V''(c) < 0$ for any $c \in [0, C_V)$, and the claim follows. ■

A.1 Proof of Proposition 1

In this section, I express the function $V(c)$ as a function of X, denoting the threshold satisfying $V'(X, X) - 1 = V''(X, X) = 0$. To prove the claim, I exploit the following auxiliary results.

Lemma 7 *The function $V(c, X)$ is decreasing in the payout threshold X.*

Proof. To prove the claim, I take $X_1 < X_2$, and I define the auxiliary function $k(c) = V(c, X_1) - V(c, X_2)$, that satisfies

$$(\rho + \Phi + \lambda)k(c) = (rc + \mu)k'(c) + 0.5\sigma^2 k''(c) + \lambda(X_1 - X_2)[r/(\rho + \Phi) - 1] \tag{30}$$

for any $c \in [0, X_1]$. By previous result and straightforward calculations, the function is positive at X_2 as $k(X_2) = (X_1 - X_2)[r/(\rho + \Phi) - 1] > 0$. By the definition of X_1 and X_2, the function $k(c)$ is decreasing and convex for $c \in [X_1, X_2)$. Therefore, $k(X_1) > 0$. Note that the function cannot have a negative local minimum on $[0, X_1]$ because the last term on the right hand side of (30) is positive. In addition, the function $k'(c)$ does not have a positive local maximum nor a negative local minimum, otherwise the equation $(\rho + \Phi + \lambda - r)k'(c) = (rc + \mu)k''(c) + 0.5\sigma^2 k'''(c)$ would not hold (respectively $k'(c) > 0 = k''(c) > k'''(c)$ and $k'(c) < 0 = k''(c) < k'''(c)$ at a positive maximum and at a negative minimum). As k is convex at X_1, this means that k' is increasing at X_1, and therefore it must be negative for any $c \in [0, X_1]$. Jointly with $k(X_1) > 0$, this means that $k(c) > 0$ for any $c \in [0, X_2]$. The claim follows. ■

Lemma 8 *For a given payout threshold X and two given $\eta_1 > \eta_2$, $V(c, X, \eta_2) > V(c, X; \eta_1)$ holds for any $c \in [0, X]$.*

Proof. I define $\Phi_i = \delta\eta_i$ with $i = 1, 2$, and the auxiliary function $h(c) = V(c, X; \eta_2) - V(c, X; \eta_1)$. I need to prove that, for a given payout threshold X, $h(c) > 0$ for any $c \in [0, X]$. At X, the function is positive as

$$h(X) = (rX + \mu)\left(\frac{1}{\rho + \Phi_2} - \frac{1}{\rho + \Phi_1}\right) = (rX + \mu)\frac{\Phi_1 - \Phi_2}{(\rho + \Phi_1)(\rho + \Phi_2)} > 0,$$

because $\Phi_1 > \Phi_2$ as $\eta_1 > \eta_2$, and $h'(X) = h''(X) = 0$. In addition, the function satisfies

$$[rc + \mu]h'(c) + \frac{\sigma^2}{2}h''(c) - (\rho + \lambda + \Phi_2)h(c) + \lambda h(X) = (\Phi_2 - \Phi_1)V(c, X; \chi_1)$$

and the right hand side is negative. Differentiating gives $[rc + \mu]h''(c) + \frac{\sigma^2}{2}h'''(c) - (\rho + \lambda + \Phi_2 - r)h'(c) = (\Phi_2 - \Phi_1)V_s'(c, X; \chi_1)$. At X, I get $\frac{\sigma^2}{2}h'''(X) = \Phi_2 - \Phi_1$, meaning that $h'''(X) < 0$. This means that the second derivative is decreasing in a neighbourhood of X, so one has $h''(c) > 0$ in a left neighbourhood of X. In turn, this means that $h'(c)$ is increasing in such a neighbourhood of X, then implying that $h'(c) < 0$ in a left neighbourhood of X. Now I need to prove that the function is decreasing for any c smaller that X. Note that, by the ODE above, $h'(c)$ cannot have a negative local minimum. As $h'(X) = 0$ and it is negative and increasing in a left neighbourhood of X, this means that $h'(c)$ should be negative for any $c < X$, so $h(c)$ is always decreasing. As it is positive at X, it means that it should be always positive, so $h(c) > h(X) > 0$ so it is positive for any $c < X$. ■

Exploiting the results above, I can prove the following lemma.

Lemma 9 *For any $\eta_1 > \eta_2$, $C_V(\eta_1) < C_V(\eta_2)$.*

Proof. The payout thresholds $C_V(\eta_1)$ and $C_V(\eta_2)$ are the unique solution to the boundary conditions $V(0, C_V(\eta_2); \eta_2) - \ell = 0 = V(0, C_V(\eta_1); \eta_1) - \ell$. Exploiting the result in Lemma 8, I now take, for instance, $X = C_V(\eta_1)$. It then follows that

$$V(0, C_V(\eta_1); \eta_2) - \ell \ > \ 0 \ = \ V(0, C_V(\eta_1); \eta_1) - \ell.$$

As V is decreasing in the payout threshold, this means that $C_V(\eta_1) < C_V(\eta_2)$ to get the equality $\ell - V(0, C_V(\eta_2); \eta_2) = 0$. The claim follows. ■

The next results stem from Lemma 9.

Corollary 10 *When the bid-ask spread is positive, the target cash level is lower than in the benchmark case with no bid-ask spread, i.e.* $C_V < C^*$.

Note also that all the results in this section can be extended for two parameters $\delta_1 > \delta_2$. The following result is then straightforward.

Corollary 11 *For any* $\delta_1 > \delta_2$, $C_V(\delta_1) < C_V(\delta_2)$.

A.1.1 Providing liquidity via share repurchases

The analysis shows that trading costs translate into a larger cost of capital, which exacerbates the firm's financial constraints. A question arises as to whether it would be optimal for the firm to commit to repurchasing shares of shocked shareholders for any $c < C_V$, then effectively serving as a liquidity provider. In so doing, the firm would then decrease its cost of capital.

Suppose that the firm follows this policy and repurchases the shares of shocked investors at their fair value, meaning that the firm bears a constant cash outflow equal to $\delta V(c)$ on any time interval. Firm value would satisfy

$$\rho V(c) = [rc + \mu - \delta V(c)]\, V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda\, [V(C_V) - C_V + c - V(c)] + \delta\, (V(c) - V(c)) \ , \quad (31)$$

subject to the same boundary conditions in Section 3.1. Equation (31) differs from (7) as the loss borne by shocked investors (the last term on the right-hand side), is $\delta\,(V(c) - V(c)) = 0$. That is, if the firm commits to repurchase the shares of shocked shareholders at fair price, there is no loss borne by shocked investors. Yet, this policy is suboptimal, as the decrease in firm value due to the flow cost of repurchasing shares of shocked shareholders (i.e., the term $\delta V(c)V'(c)$ on the right-hand side of this equation) is larger than the decrease in firm value due to the higher return required by investors (i.e., the term $\delta V(c)\eta$ in equation (7)). The reason is that, for any $c < C_V$, the marginal value of cash is greater inside the firm than if paid out ($V'(c) \geq 1 > \eta > 0$; see Lemma 6 in Appendix A).

Alternatively, the firm could buy back the shares of shocked shareholders at a price smaller than $\delta V(c)$. Yet, this price cannot be smaller than $\delta V(c)(1 - \eta)$—otherwise shocked shareholders would be better off selling the stock to trading firms than to the firm. That is, the firm needs to buy back shares at a price at least equal to $\delta V(c)(1 - \eta)$. When following this policy, firm value would satisfy[28]

$$(\rho + \delta\eta)V(c) = [rc + \mu - (1-\eta)\delta V(c)]\, V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda\, [V(C_V) - C_V + c - V(c)]\,.$$

Again, this policy decreases firm value compared to that derived in Section 3.1. In fact, the firm would need to bear the greater return required by the investors as well as to commit to repurchase shares for $c < C_V$, then draining the firm's cash flows.

This analysis suggests that share repurchases are suboptimal when cash reserves are below the target level C_V.[29] The reason is that, for any $c < C_V$, cash is more valuable inside the firm ($V'(c) > 1$) than if paid out. Instead of repurchasing shares for any level of cash reserves c, the optimal policy prescribes that the firm should "provide liquidity" to shareholders by reducing the target cash level with respect to the benchmark case with zero bid-ask spread (as from Proposition 1), meaning that the payout threshold is hit more often (as shown by Proposition 2).

A.2 Proof of Proposition 2

Using the insights from Dixit and Pindyck (1994), the dynamics of $P_p(c, X)$ are given by

$$P_p'(c)(rc + \mu) + \frac{\sigma^2}{2} P_p''(c) - \lambda\, P_p(c) = 0$$
$$\text{s.t. } P_p(0) = 0, \quad P_p(X) = 1.$$

The first boundary condition implies that when the controlled cash process is absorbed at zero, the firm liquidates and the payout probability is zero. The second boundary condition is obvious given that cash is paid out at X. The following lemma shows that greater bid-ask spreads are associated with larger payout probability.

Lemma 12 *For any $\eta_1 > \eta_2$, $P_p(c, C_V(\eta_1)) \geq P_p(c, C_V(\eta_2))$.*

Proof. By Lemma 9, $C_V(\eta_1) < C_V(\eta_2)$. To ease the notation throughout the proof, I define $X_1 \equiv C_V(\eta_1)$ and $X_2 \equiv C_V(\eta_2)$. Consider the function

$$h(c) = P_p(c, X_1) - P_p(c, X_2).$$

Because of the boundary conditions at zero and X_1, $h(0) = 0$ and $h(X_1) = 1 - P_p(c, X_2) > 0$. This means that the function is null at zero, and positive at C_V. Note that $h(c)$ cannot have neither a positive local maximum ($h(c) > 0$, $h'(c) = 0$, $h''(c) < 0$) nor a negative local minimum ($h(c) < 0$, $h'(c) = 0$, $h''(c) > 0$) on $[0, X_1]$, as otherwise the equation $h''(c)\frac{\sigma^2}{2} + h'(c)[rc + \mu] - \lambda h(c) = 0$ would not hold. Therefore, the function must be always positive and increasing over the relevant interval, and the claim follows. ■

[28]The term $\delta\eta V(c)$ on the left-hand side stems from the fact that the loss borne by shocked shareholders under this policy is $(1 - \eta - 1)\delta V(c) = -\eta\delta V(c)$. I.e., this equation is equivalent to $\rho V(c) = [rc + \mu - (1-\eta)\delta V(c)]\, V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda\, [V(C_V) - C_V + c - V(c)] + (1 - \eta - 1)\delta V(c)$, where the last term on the right-hand side is the loss borne by shocked shareholders that are repurchased at price $\delta V(c)(1-\eta)$.

[29]It is then possible to generalize that repurchasing shares is suboptimal for any $c < C_V$ and at any price in the interval $[\delta V(c)(1-\eta), \delta V(c)]$.

The result below is a straightforward consequence of Lemma 12 and the fact that, in the absence of trading costs, $\eta = 0$ (or $\delta = 0$).

Corollary 13 *When trading the firm's stock is costly, the payout probability P_p is larger than in the benchmark case with no trading costs, i.e. $P_p(c, C^*) < P_p(c, C_V)$.*

These results can be extended for two parameters $\delta_1 > \delta_2$, as follows.

Corollary 14 *For any $\delta_1 > \delta_2$, $P_p(c, C_V(\delta_1)) \geq P_p(c, C_V(\delta_2))$.*

A.3 Proof of Proposition 3

I derive the results regarding the probability of liquidation $P_l(c, X)$, because the probability of external financing is just $P_f(c, X) = 1 - P_l(c, X)$. Using standard methods (see e.g., Dixit and Pyndick, 1994), the dynamics of $P_l(c, X)$ are given by

$$P_l'(c)(rc + \mu) + \frac{\sigma^2}{2} P_l''(c) - \lambda \; P_l(c) = 0$$

$$\text{s.t.} \;\; P_l(0) = 1 \tag{32}$$

$$P_l'(X) = \; 0, \tag{33}$$

where the first boundary condition is given by the definition of P_l, while the second boundary condition is due to reflection at the payout threshold.

Now I prove that the probability of liquidation is higher when the firm's stocks are illiquid (i.e, the bid-ask spread associated with the firm stock is positive). To do so, I first prove that the probabilities $P_l(c, C^*)$ and $P_l(c, C_V)$ are decreasing and convex in c. In the following, I employ the generic function $P_l(c, X) \equiv P_l(c)$.

Lemma 15 *The probability $P_l(c, X)$ is decreasing and convex for any $c \in [0, X]$.*

Proof. To prove the claim, I exploit arguments analogous to those of Lemma 6. As $P_l'(X) = \; 0$ and $P_l(X) \geq 0$, it must be that $P_l''(X) > 0$. Then, there exists a left neighbourhood of X, $[X - \epsilon, X]$ with $\epsilon > 0$, over which $P_l'(c) < 0$ and $P_l''(c) > 0$. Toward a contradiction, suppose that there exists some $c \in [0, X - \epsilon]$ where $P_l'(c) > 0$. Then, there should be a $\bar{C}$ such that $P_l'(\bar{C}) = \; 0$, while $P_l'(c) < 0$ for $c \in [\bar{C}, X]$. For any $c \in [\bar{C}, X]$ it must be that

$$P_l''(c) = \; \frac{2}{\sigma^2} \left[\lambda P_l(c) - P_l'(c)(rc + \mu) \right] > \frac{2}{\sigma^2} \; \lambda P_l(X) \; > \; 0 \, .$$

Then, $P_l''(c) > 0$ for any $c \in [\bar{C}, X]$ means that $P_l'(c)$ is always increasing on $c \in [\bar{C}, X]$, contradicting $P_l'(\bar{C}) = \; P_l'(X) = 0$. The claim follows. ■

Now I prove that $P_l(c, C_V) \geq P_l(c, C^*)$.

Lemma 16 *For any $\eta_1 > \eta_2$, $P_l(c, C_V(\eta_1)) \geq P_l(c, C_V(\eta_2))$.*

Proof. By Lemma 9, $C_V(\eta_1) < C_V(\eta_2)$. To ease the notation throughout the proof, I define $X_1 \equiv C_V(\eta_1)$ and $X_2 \equiv C_V(\eta_2)$. By Lemma 15, the functions $P_l(c, X_1)$ and $P_l(c, X_2)$ are positive, decreasing and convex over the interval of definition. I define the auxiliary function

$$h(c) = P_l(c, X_1) - P_l(c, X_2).$$

Note that $h(c)$ cannot have neither a positive local maximum ($h(c) > 0$, $h'(c) = 0$, $h''(c) < 0$) nor a negative local minimum ($h(c) < 0$, $h'(c) = 0$, $h''(c) > 0$) on $[0, X_1]$, as otherwise the equation $h''(c)\frac{\sigma^2}{2} + h'(c)[rc + \mu] - \lambda h(c) = 0$ would not hold. In addition, $h(0) = 0$, and $h'(X_1) = -P_l'(c, X_2) > 0$ because of the boundary conditions at zero and at X_1. This means that the function is null at the origin, and increasing at C_V. Toward a contradiction, assume that $h(X_1)$ is negative. This would imply the existence of a negative local minimum, given that the function is null at zero and it is increasing at X_1. This cannot be the case as argued above, contradicting that $h(X_1) < 0$. Therefore, the function must be always positive, and the claim follows. ■

The result below is a straightforward consequence of Lemma 16 and the fact that, in the absence of trading costs, $\eta = 0$ (or $\delta = 0$).

Corollary 17 *When the bid-ask spread associated with the firm's stock is positive, the probability of liquidation P_l is larger than in the case in which the bid-ask spread is zero, i.e. $P_l(c, C^*) < P_l(c, C_V)$.*

These results can be extended to the case in which the parameter δ is varied.

Corollary 18 *For any $\delta_1 > \delta_2$, $P_l(c, C_V(\delta_1)) \geq P_l(c, C_V(\delta_2))$.*

A.4 Proof of Proposition 4

I exploit the dynamic programming result in Décamps and Villeneuve (2007) and HMM, establishing that the growth option has a non-positive NPV if and only if $V(c) > V_+(c - I)$ for any $c \geq 0$, where I denote by $V_+(c - I)$ the value of the firm after investment. To prove the claim, I rely on the following lemma.

Lemma 19 *$V(c) \geq V_+(c - I)$ for any $c \geq I$ if and only if $I \geq I_V$, where I_V satisfies the expression reported in Proposition 4.*

Proof. I define $\bar{c} = \max[C_V, I + C_{V+}]$. The inequality $V(c) \geq V_+(c - I)$ for $c > \bar{c}$ means that $c - C_V + V(C_V) \geq c - C_{V+} - I + V_+(C_{V+})$. Using the definition of I_V, the former inequality is equivalent to the inequality $I \geq I_V$, by straightforward calculations.

To prove the sufficient condition, I can just prove that $V(c) \geq V_+(c - I_V)$ for any $c \geq I_V$. I exploit the inequalities $C_V < C_{V+} + I_V$ and $\mu_+ - \mu - rI_V > 0$ (these inequalities stem from a slight modification of Lemma C.3 in HMM, so I omit the details). For $c \geq C_V$, the following inequality

$$V_+(c - I_V) \leq V_+(C_{V+}) + c - I_V - C_{V+} = c - C_V + V(C_V) = V(c)$$

holds. The first inequality is due to the concavity of V_+. The first equality is given by the definition of I_V, whereas the second equality is due to the linearity of V above C_V. I now need to prove the result for $c \in [I_V, C_V]$. To this end, I define the auxiliary function $u(c) = V(c) - V_+(c - I_V)$. The function $u(c)$ is positive at C_V as argued above, $u'(C_V) < 0$ and $u''(C_V) > 0$. On the interval of interest it satisfies

$$(\rho + \Phi + \lambda)u(c) = (rc + \mu)u'(c) + \frac{\sigma^2}{2}u''(c) + (\mu + rI_V - \mu_+)V'_+(c - I_V) \\ + \lambda(V(C_V) - C_V - V_+(C_{V+}) + C_{V+} + I_V)$$

where the last term on the right hand side is zero by the definition of I_V, while the third term is negative. Then, the function cannot have a positive local maximum here, because otherwise $u(c) > 0$, $u''(c) < 0 = u'(c)$, and the ODE above would not hold. Jointly with the fact that $u(C_V)$ is positive, decreasing and convex means that the function is always decreasing on this interval. Then, $u(c)$ is also always positive, and the claim holds. ■

B Proof of the Results in Section 4.1

Two separate cases are considered, depending on the relative magnitude of participation or funding costs borne by liquidity providers.

Case $\eta(c) < \chi$ for any c. This is the case if participation and funding costs are sufficiently low, so that the following inequality

$$\frac{\gamma}{\delta(\chi - \kappa)} \leq \ell \tag{34}$$

holds. When this is the case, firm value satisfies equation (16) for any $c < C_V$, and firm value is solved subject to the boundary condition at the liquidation threshold and at C_V as reported in the main text. As a straightforward extension of Lemma 6, it is possible to show that $V(c)$ is increasing and concave in cash reserves.

Case $\eta(c) = \chi$ for some c. This is the case if participation and funding costs are sufficiently large, so that condition (34) does not hold. Because firm value is increasing in c, there is at most one cash threshold $\underline{C} \in [0, C_V]$ such that the proportional bid-ask spread is $\eta(c)$ lower than χ for $c \in [\underline{C}, C_V]$ and solves equation (13). For $c \in [0, \underline{C})$, a fraction δ of shareholders is indifferent between keeping the stock or selling it at price $\chi V(c)$. Continuity and smoothness at $\underline{C}$ mean that the system of equations (16) and (17) is solved subject to the following conditions:

$$\lim_{c \uparrow \underline{C}} V(c) = \lim_{c \downarrow \underline{C}} V(c) \quad \text{and} \quad \lim_{c \uparrow \underline{C}} V'(c) = \lim_{c \downarrow \underline{C}} V'(c)$$

on top of the boundary conditions at the liquidation and payout threshold ($V(0) - \ell = \lim_{c \uparrow C_V} V'(c) - 1 = \lim_{c \uparrow C_V} V''(c) = 0$). In this case too, I show next that the value function is strictly monotone and concave over $0 \leq c < C_V$. Differentiating equation (16) gives the following ODE

$$(\rho + \lambda + \delta\kappa - r)\, V'(c) = V''(c)\,(rc + \mu) + \frac{\sigma^2}{2}V'''(c) + \lambda.$$

Jointly with the boundaries $V'(C_V) = 1$ and $V''(C_V) = 0$, this ODE implies that $V'''(C_V) > 0$, meaning that there exists a left neighborhood of C_V such that for any $c \in (C_V - \epsilon, C_V)$, with $\epsilon > 0$, the inequalities $V'(c) > 1$ and $V''(c) < 0$ hold. Toward a contradiction, I assume that $V'(c) < 1$

for some $c \in [0, C_V - \epsilon]$. Then, there should be a point $C_c \in [0, C_V - \epsilon]$ such that $V'(C_c) = 1$ and $V'(c) > 1$ over (C_c, C_V), so $V(C_V) - V(c) > C_V - c$ for any $c \in (C_c, C_V)$. The point C_c could belong either to the interval $[0, \underline{C}]$ or in the interval $[\underline{C}, C_V]$. I now discriminate between these two cases. If $\underline{C} < C_c < C_V$, it must be that for any $c \in (C_c, C_V)$

$$V''(c) = \frac{2}{\sigma^2} \left\{ (\rho + \lambda + \delta\kappa) V(c) - (rc + \mu)V'(c) + \gamma - \lambda[V(C_V) + c - C_V] \right\}.$$

Using $V(C_V) - V(c) > C_V - c$, jointly with $V(C_V) = \frac{rC_V + \mu - \gamma}{\rho + \delta\kappa}$, it follows that

$$V''(c) < \frac{2}{\sigma^2} \left\{ (\rho + \delta\kappa)(V(C_V) + c - C_V) - rc - \mu + \gamma \right\} = \frac{2}{\sigma^2}(c - C_V)(\rho + \delta\kappa - r) < 0.$$

This means that $V'(c)$ is decreasing for any $c \in (C_c, C_V)$, which contradicts $V'(C_c) = V'(C_V) = 1$. So, $V'(c) > 1$ for any $c \in [\underline{C}, C_V)$, so such C_c does not exist on $[\underline{C}, C_V]$.

I now consider the case $0 < C_c < \underline{C}$. Should such point C_c exist, the strict concavity of $V(c)$ over $[\underline{C}, C_V]$ means that there should be a maximum $C_m \in [C_c, \underline{C}]$ for the first derivative over the interval $(C_c, \underline{C})$, such that $V'(C_m) > 1$, $V''(C_m) = 0$ and $V'''(C_m) < 0$. Differentiating equation (17) gives

$$V''(c)[rc + \mu] + V'''(c)\frac{\sigma^2}{2} - V'(c)(\rho + \delta\chi - r) + \lambda(1 - V'(c)) = 0.$$

Then, $V'''(C_m)\frac{\sigma^2}{2} = (\rho + \delta\chi - r)V'(C_m) + \lambda(V'(C_m) - 1) > 0$, which contradicts the existence of such a maximum C_m for $V'(c)$. It follows that C_c cannot exists, and the claim follows.

Investment Decision with Endogenous Liquidity Provision. Following arguments similar to those in Appendix A.4, the firm finds it optimal to invest in the growth option if it has positive NPV. This is the case if $V(c) > V_+(c - I)$, where again I denote by $V_+(c - I)$ the value of the firm after investment. Thus, a straightforward modification of Lemma 19 implies that the zero-NPV investment (i.e., that makes the NPV of the project equal to zero) is given by the following expression:

$$I_V = \frac{\mu_+ - \mu}{\rho + \delta\kappa} - \left(1 - \frac{1}{\rho + \delta\kappa}\right)(C_{V+} - C_V) \tag{35}$$

where I denote by C_{V+} the target cash level after investment (i.e., when the cash flow drift is μ_+).

B.1 Proof of the Result in Section 4.3

Consider the zero-profit condition if the firm enters the DMM contract. The equilibrium payment Γ needs to guarantee that the maximum contractual bid-ask spread is $\bar{\eta} < \chi$. As shown in the analysis in Section 4.1, the bid-ask spread decreases as firm value increases. Thus, the minimum Γ that the firm needs to pay to the DMM solves the following equation:

$$\delta V(0)(1 - \kappa) - \delta(1 - \bar{\eta})V(0) + \Gamma = \gamma, \tag{36}$$

which gives $\Gamma^* = \gamma - \delta(\bar{\eta} - \kappa)V(0)$. In this environment, the bid-ask spread is given by the following expression:

$$\eta(c) = \frac{\gamma - \Gamma^*}{\delta V(c)} + \kappa \tag{37}$$

which indeed is lower than the equilibrium bid-ask when the firm does not enter the DMM contract. Next, I prove Proposition 5, showing that, on net, entering the DMM contract cannot increase firm value.

Proof. Comparing equation (19) with equation (16) illustrates that the two equations only differ for the additional term $-\Gamma^* V'(c) + \Gamma^*$ on the right-hand side. This expression can be rewritten as $\Gamma^*(1 - V'(c))$. As $V'(c) \geq 1$ for any $c < C_V$, the expression $\Gamma^*(1 - V'(c))$ is strictly negative for any $c < C_V$, which implies that it is suboptimal for a financially constrained firm to subsidize the DMM. ■

C Proof of the Results in Section 5

C.1 Proof of the Results in Section 5.2

This section reports additional calculations for the alternative setup reported in Section 5.2, investigating an environment with endogenous capital accumulation and continuous investment. Using standard arguments, firm value satisfies the following equation:

$$\rho V(C,K) = \max_i (i-d)KV_K + \left[\left(\mu - i - \frac{\theta i^2}{2}\right)K + rC\right]V_C + \frac{1}{2}K^2\sigma^2 V_{CC} \tag{38}$$

$$+ \delta\left[\Phi(C,K) - V(C,K)\right] + \lambda\left[V(C^*,K) - C^* - V(C,K) + C\right] \tag{39}$$

where $\delta\Phi(C,K)$ denotes the aggregate claim of shocked shareholders in this extended setup. Conjecture that $V(C,K) = v(w)K$, where $w = C/K$ denotes scaled cash reserves. Also, let us denote by W^* the target scaled cash reserves. Substituting in the above equation gives

$$(\rho + \delta\eta)v(w) = \max_i (i-d)(v - wv') + \left[\mu - i - \frac{\theta i^2}{2} + rw\right]v' + \frac{\sigma^2}{2}v''$$
$$+ \lambda\left[v(W^*) - W^* - v(w) + w\right] \tag{40}$$

Differentiating with respect to i gives the expression for the optimal investment rate

$$i(w) = \frac{v(w) - v'(w)(1+w)}{\theta v'(w)},$$

as reported in the main text. Substituting the expression for $i(w)$ into equation (40) and imposing boundary conditions similar to those in Section 3.1 gives scaled firm value. As in BCW, denote by l the liquidation value of assets scaled by the capital stock. Thus, firm value satisfies the condition $v(0) = l$ when scaled cash reserves reach zero. Moreover, at the threshold W^*, the following boundary conditions hold: $v'(W^*) = 1$ and $v''(W^*) = 0$.

Figure 5 analyzes scaled firm value and the optimal investment rates for different values of η, as reported in the legend. On top of the parameters specific to firm's financing frictions (λ) and to secondary market transactions (δ, χ)—for which I use the values reported in Table I—I use the parameters pertaining to the neoclassic q framework reported in Table 1 of BCW. Specifically, the adjustment-cost parameter is set to 1.5, the rate of depreciation is equal to 10.07%, the liquidation value of capital is set to 0.9, the mean and the volatility of the productivity shock are respectively 0.18 and 0.09, the risk-free is equal to 0.06, and the opportunity cost of cash is equal to 0.01.

C.2 Proof of the Results in Section 5.4

For any $c < \underline{C}_\theta$, shocked shareholders bear the cost $\delta\chi V(c)$ on aggregate (as they are not able to sell the stock to trading firms). Firm value satisfies the following ODE:

$$(\rho + \delta\chi)\, V(c) = \ (rc + \mu)\, V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda\left[V(C_V) - C_V + c - V(c)\right]. \tag{41}$$

This equation admits an interpretation analogous to (7), but the return required by the investors (the left-hand side) is larger and equal to $\rho + \delta\chi$. The reason is that declining liquidity provision in the market of the stock increases trading costs to shocked shareholders, and so increases the illiquidity premium and the firm's cost of capital.

For any $c > \underline{C}_\theta$, the expected loss associated with a liquidity shock is given by the following expression: $\left[\pi(c)\eta\delta V(c) + \left(1 - \pi(c)\right)\chi\delta V(c)\right] dt$. The first term represents the probability-weighted loss when selling the stock to an active trading firm, whereas the second term represents the probability-weighted loss associated with keeping the stock. For any $c \geq \underline{C}_\theta$, firm value satisfies

$$(\rho + \delta\eta)\, V(c) = (rc + \mu)\, V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda\left[V(C_V) - C_V + c - V(c)\right] - \frac{\alpha\gamma(\chi - \eta)}{(\eta - \kappa)}. \tag{42}$$

Equation (42) shows that the return required by the investors (the left-hand side) is the same as equation (7). Yet, the last term on the right-hand side reveals that intermediaries' participation frictions matter to shareholders. These frictions hamper liquidity provision in the market of the stock and so increase the expected trading costs incurred by shocked shareholders. This term is akin to a flow cost, which is larger if intermediaries' participation frictions, α and γ, increase.

Continuity and smoothness at $\underline{C}_\theta$ mean that the system of equations (41)–(42) is solved subject to the following conditions:

$$\lim_{c \uparrow \underline{C}_\theta} V(c) = \lim_{c \downarrow \underline{C}_\theta} V(c) \quad \text{and} \quad \lim_{c \uparrow \underline{C}_\theta} V'(c) = \lim_{c \downarrow \underline{C}_\theta} V'(c)\,.$$

In addition, $V(c)$ satisfies $V(0) - \ell = \ \lim_{c \uparrow C_V} V'(c) - 1 = 0$ at the liquidation threshold and at the target cash level, similar to Section 3.1. Lastly, the target cash level is identified by the super-contact condition, $\lim_{c \uparrow C_V} V''(c) = \ 0$.

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Symbol	Description	Value
	FIRM	
ρ	Risk-free rate	0.02
r	Return on cash	0.01
μ	Cash flow drift	0.05
μ_+	Post-investment cash flow drift	0.06
σ	Cash flow volatility	0.12
ϕ	Recovery rate in liquidation	0.55
λ	Arrival rate of financing opportunities	0.75
L	Credit line limit	0.08
β	Credit line spread	0.015
	STOCK TRANSACTIONS	
δ	Arrival rate of liquidity shocks	0.70
χ	Loss due to liquidity shocks	0.02
κ	Liquidity providers' funding cost	0.003
γ	Liquidity providers' participation cost	0.007
$\bar{\eta}$	DMM's bid-ask cap	0.008

TABLE I: BENCHMARK PARAMETERS.

Bid-ask spread (Basis points)	Target Cash Level	Financing Probability	Liquidation Probability	Payout Probability	Zero-NPV investment cost	Firm value
0	0.546	87.2%	12.8%	24.9%	0.504	2.773
10	0.534	86.9%	13.1%	25.4%	0.487	2.673
20	0.522	86.6%	13.4%	25.9%	0.471	2.580
30	0.510	86.3%	13.7%	26.4%	0.455	2.493
40	0.498	86.0%	14.0%	26.9%	0.441	2.411
50	0.486	85.6%	14.4%	27.5%	0.427	2.334
60	0.473	85.2%	14.8%	28.0%	0.414	2.262
70	0.460	84.8%	15.2%	28.7%	0.401	2.193
80	0.447	84.3%	15.7%	29.3%	0.389	2.128
90	0.432	83.8%	16.2%	30.1%	0.378	2.066
100	0.417	83.2%	16.8%	30.9%	0.366	2.006
120	0.383	81.6%	18.4%	32.9%	0.345	1.895
140	0.340	79.2%	20.8%	35.5%	0.323	1.792
160	0.285	74.7%	25.3%	39.4%	0.303	1.694
180	0.214	64.9%	35.1%	44.6%	0.285	1.599

Table II: Effect of bid-ask spreads on corporate outcomes.

The table reports the target cash level, the average probability of external financing, of liquidation, and of payout, the zero-NPV investment cost, and firm value (at the target cash level C_V) when varying the bid-ask spread, as reported in the first column.

Bid-ask spread (Basis points)	$C_V/2$	$C_V/4$	$C_V/8$
0	1.97%	14.1%	37.6%
10	2.15%	14.7%	38.5%
20	2.34%	15.4%	39.3%
30	2.56%	16.1%	40.1%
40	2.79%	16.8%	41.0%
50	3.06%	17.5%	41.9%
60	3.35%	18.3%	42.9%
70	3.69%	19.2%	43.9%
80	4.08%	20.2%	45.0%
90	4.53%	21.2%	46.1%
100	5.08%	22.5%	47.4%
120	6.58%	25.4%	50.5%
140	9.09%	29.7%	54.5%
160	14.0%	36.6%	60.3%
180	25.3%	48.4%	69.2%

TABLE III: BID-ASK SPREADS AND PROBABILITY OF FORCED LIQUIDATION.

The table reports the firm's probability of liquidation at different levels of cash reserves (i.e., at $C_V/2$, $C_V/4$, and $C_V/8$) when varying the bid-ask spread (as reported in the first column).

Participation friction	Target cash level (Δ%)	Liquidation probability (Δ)	Payout probability (Δ)	Zero-NPV investment cost (Δ%)	Firm value (Δ%)
$\gamma = 0.1\%$	-7.44%	1.04%	1.67%	-9.86%	-11.80%
$\gamma = 0.3\%$	-9.43%	1.35%	2.14%	-10.19%	-15.24%
$\gamma = 0.5\%$	-11.77%	1.73%	2.73%	-10.63%	-18.71%
$\gamma = 0.7\%$	-14.60%	2.22%	3.46%	-11.20%	-22.23%
$\gamma = 0.9\%$	-18.17%	2.89%	4.42%	-12.03%	-25.81%
$\kappa = 0.1\%$	-8.84%	1.25%	2.00%	-4.54%	-16.42%
$\kappa = 0.2\%$	-11.66%	1.71%	2.70%	-7.95%	-19.41%
$\kappa = 0.3\%$	-14.60%	2.22%	3.46%	-11.20%	-22.23%
$\kappa = 0.4\%$	-17.73%	2.80%	4.30%	-14.35%	-24.88%
$\kappa = 0.5\%$	-21.09%	3.49%	5.25%	-17.41%	-27.40%

TABLE IV: LIQUIDITY PROVIDERS' PARTICIPATION FRICTIONS AND CORPORATE OUTCOMES.

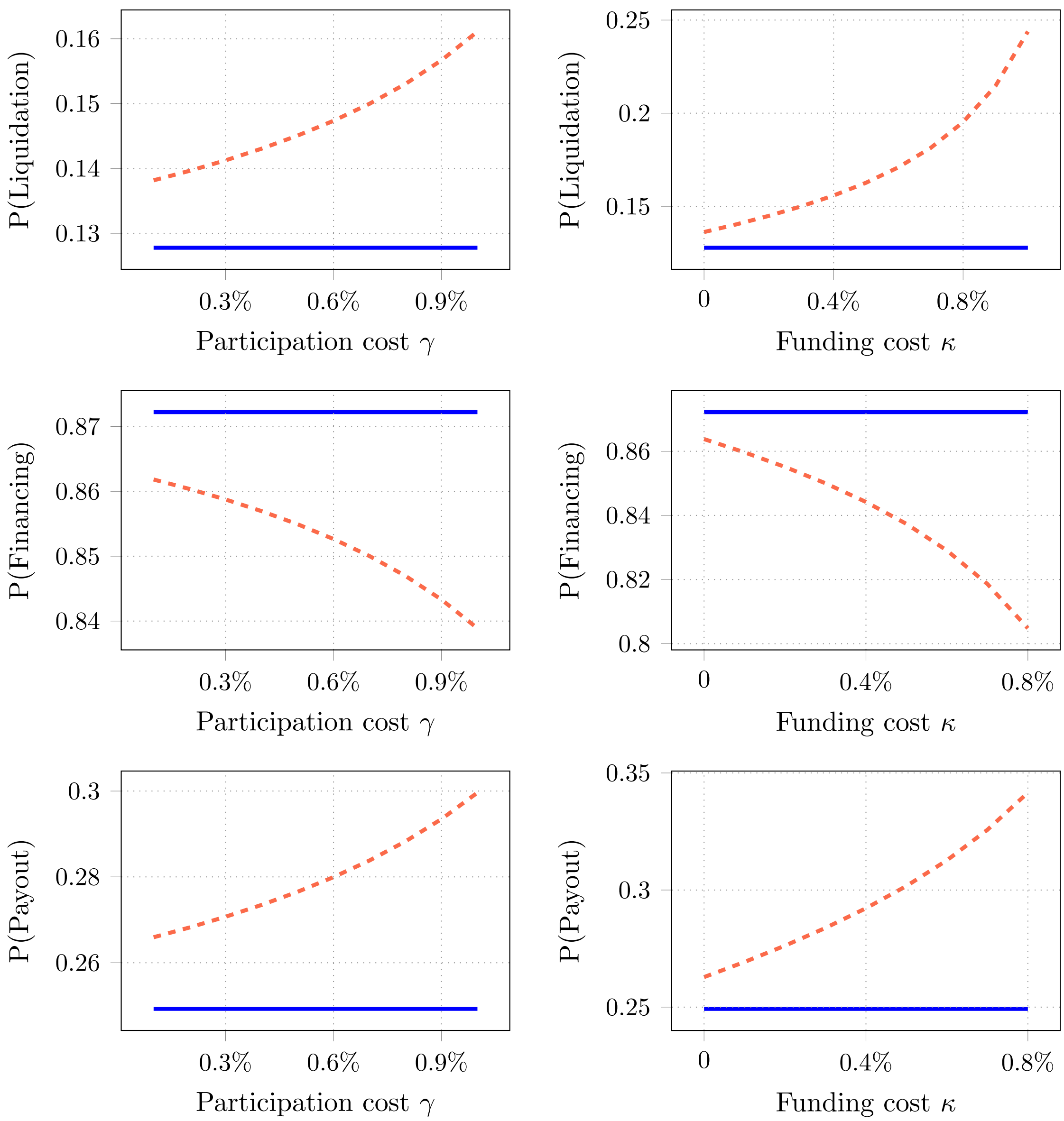
The table reports the change in the target cash level, in the probability of liquidation, in the probability of payout, in the zero-NPV investment cost, and in firm value (calculated at the target cash level) when accounting for participation fees (γ) and funding costs (κ) borne by liquidity providers with respect to a benchmark environment with no trading frictions (in which the bid-ask spread is zero).

Bid-ask spread (Basis points)	Target Cash Level	Issuance Size	Zero-NPV investment cost
$\epsilon = 0.06$, $\omega = 0.01$			
0	0.350	0.133	0.509
20	0.338	0.131	0.476
40	0.327	0.129	0.447
60	0.317	0.127	0.422
80	0.309	0.125	0.399
100	0.301	0.124	0.379
120	0.294	0.122	0.360
140	0.288	0.121	0.344
160	0.282	0.120	0.328
180	0.276	0.118	0.314
$\epsilon = 0.1$, $\omega = 0.03$			
0	0.438	0.178	0.513
20	0.424	0.175	0.480
40	0.411	0.173	0.452
60	0.401	0.171	0.426
80	0.391	0.169	0.403
100	0.382	0.167	0.383
120	0.374	0.165	0.365
140	0.367	0.164	0.348
160	0.360	0.162	0.333
180	0.354	0.161	0.319

TABLE V: EFFECT OF BID-ASK SPREADS WHEN FINANCING FRICTIONS ARE MODELED AS ISSUANCE COSTS.

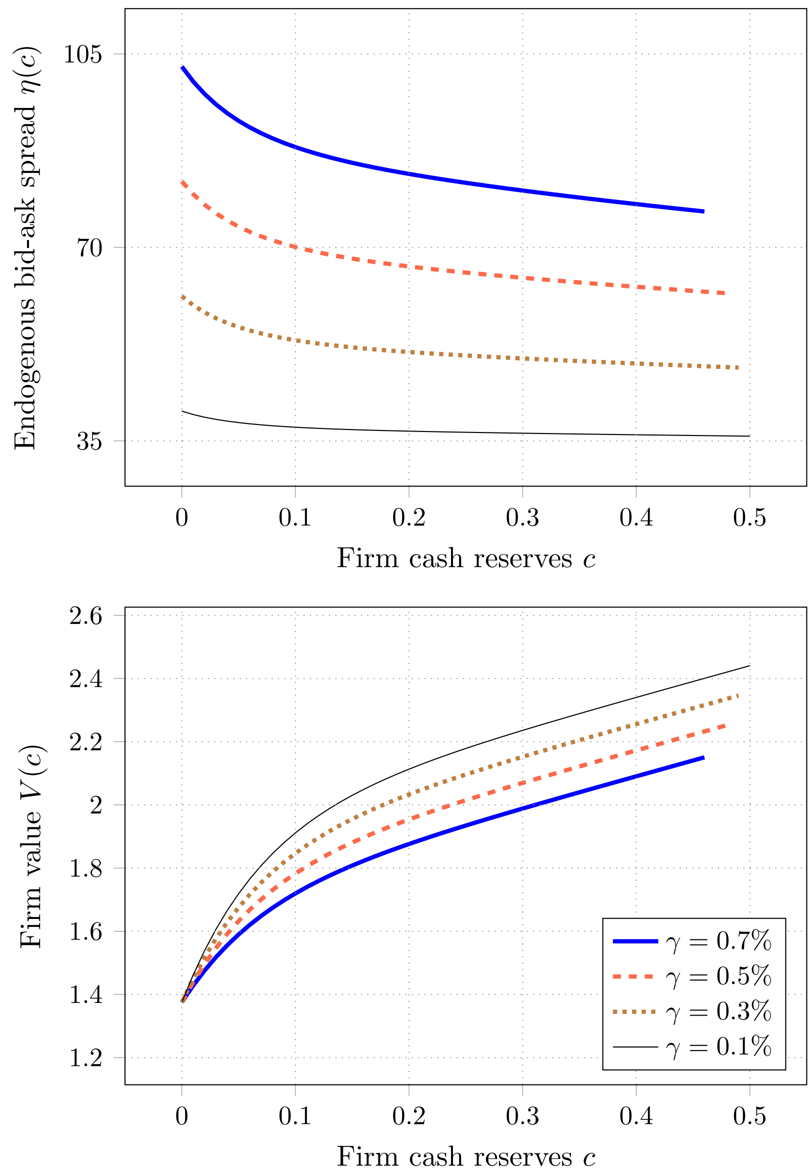
The table reports the target cash level, the size of equity issuances, and the zero-NPV investment cost when varying the bid-ask spread (as reported in the first column). Proportional and fixed financing costs are respectively equal to $\epsilon = 0.06$ and $\omega = 0.01$ in the top panel, and equal to $\epsilon = 0.1$ and $\omega = 0.03$ in the bottom panel.

Figure 1: The impact of participation frictions on the firm probability of liquidation, financing, and payout.



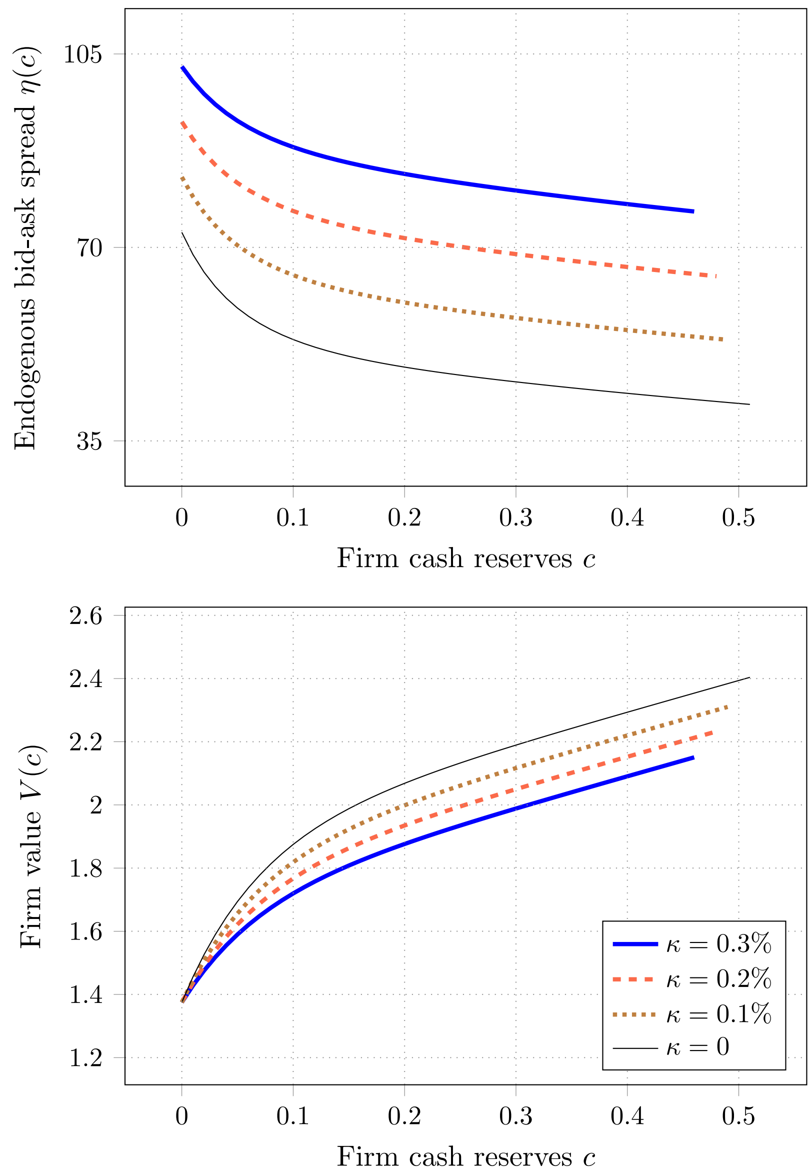
The figure shows the probability of liquidation (first panel), the probability of external financing (middle panel), and the probability of payout (bottom panel) as a function of the participation cost γ (left panel) and the funding cost κ (right panel) borne by liquidity providers. The blue solid line refer to the benchmark environment in which shareholders face no bid-ask spread when trading the stock, whereas the red dashed lines refer to the case in which the bid-ask spread is endogenously set by liquidity providers.

Figure 2: Endogenous liquidity provision and firm value when liquidity providers face participation frictions (I).



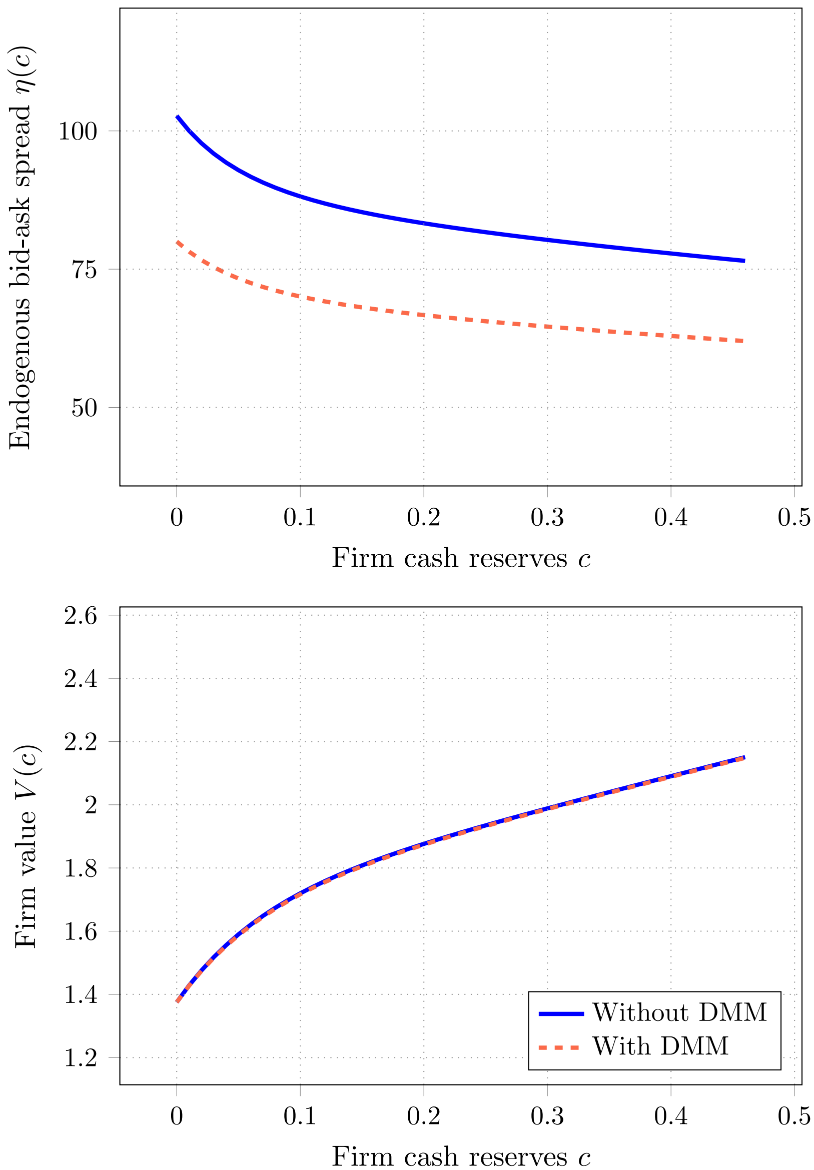
The figure shows the endogenous bid-ask spread (in basis points) as well as firm value as a function of the firm cash reserves c when varying the participation fee γ borne by liquidity providers.

Figure 3: Endogenous liquidity provision and firm value when liquidity providers face participation frictions (II).



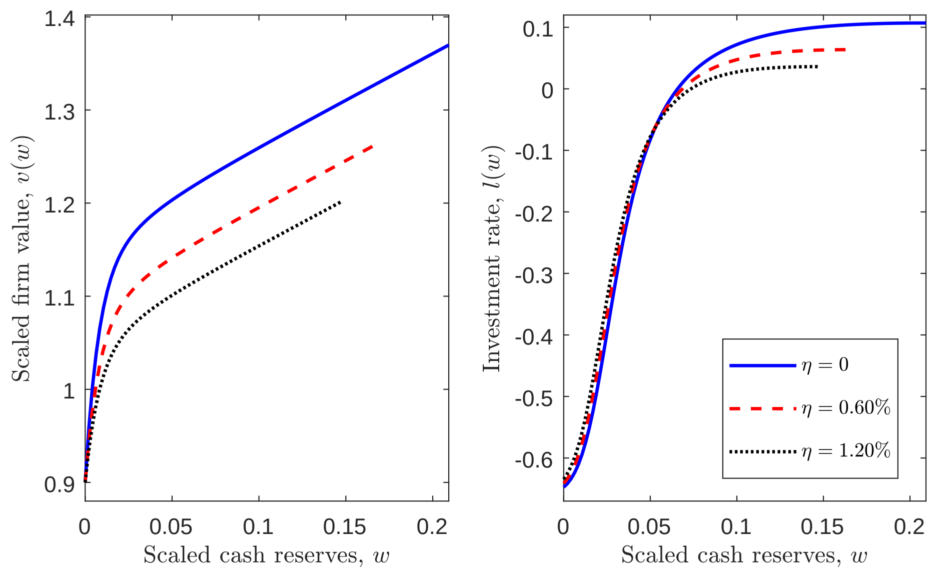
The figure shows the endogenous bid-ask spread (in basis points) as well as firm value as a function of the firm cash reserves c when varying the funding cost κ faced by liquidity providers.

Figure 4: Firm-funded designated market makers (DMM).



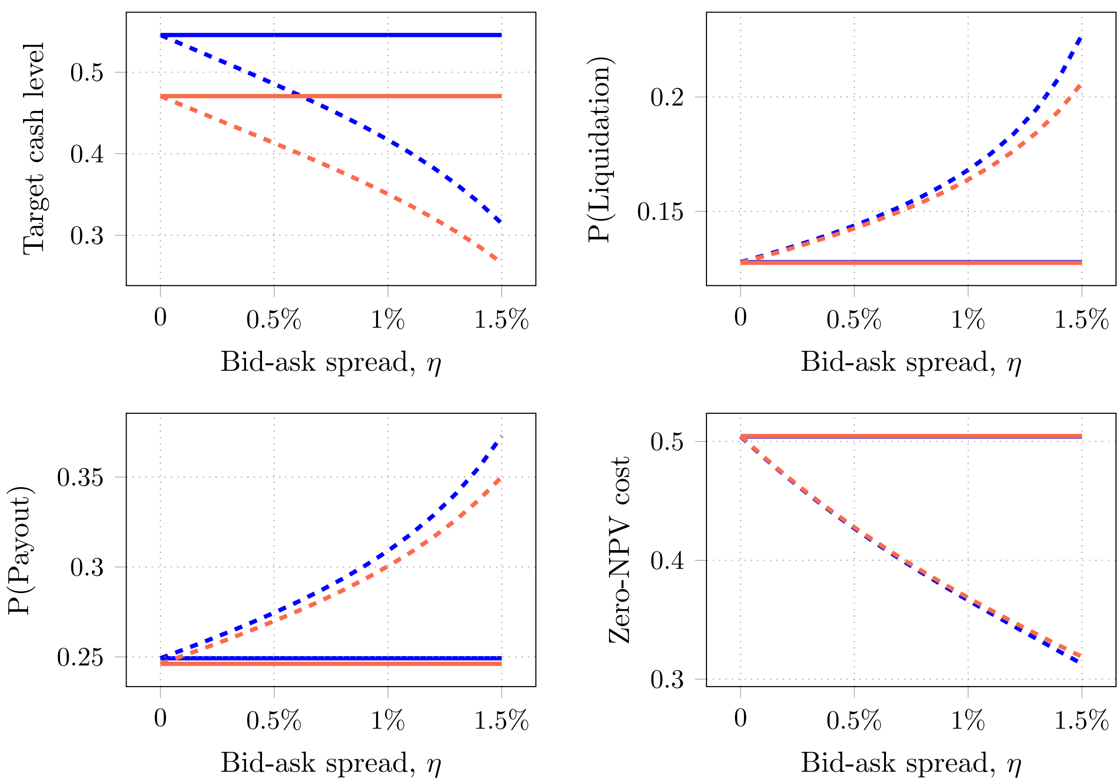
The figure shows the endogenous bid-ask spread (in basis points) as well as firm value as a function of the firm cash reserves c. The solid blue line depicts the case in which the firm does not enter the contract with the designated market maker (DMM), whereas the dashed red line depicts the environment in which the firm enters the DMM contract.

Figure 5: Continuous investment.



The left panel shows firm value scaled by capital as a function of scaled cash reserves, whereas the right panel shows the firm's optimal investment rate as a function of scaled cash reserves. The different lines correspond to different assumptions regarding the magnitude of the stock's bid-ask spread.

Figure 6: Allowing for credit line availability.



The figure shows the target level of cash reserves, the probability of liquidation, the probability of payout, and the maximum investment cost as a function of the bid-ask spread (η). The blue lines refer to a firm with no access to bank credit and when the bid-ask spread is zero (solid line) or positive (dashed line). The red lines refer to a firm having access to bank credit and when the bid-ask spread is zero (solid line) or positive (dashed line).