Reinforcing constraints*

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Abstract

Small firms face financing frictions, and trading their stocks entails non-negligible bid-ask spreads. I develop a model that studies the intertwined relation of these characteristics and the ensuing effects on corporate policies. The model shows that bid-ask spreads increase not only the cost of external financing but also the cost of internal funds, leading to smaller cash reserves and larger payouts. As a result, firm's financial constraints tighten, liquidation risk increases, investment decreases, and firm value declines. These results are reinforced when liquidity provision in the market of the stock is endogenous.

Keywords: Financial constraints, transaction costs, real effects of financial markets, small firms

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1 Introduction

Small and micro firms represent more than 80% of U.S. firms over the past forty years (see, for instance, Hou, Xue, and Zhang, Forthcoming). Two key attributes characterize these firms. First, they are largely financially constrained. Small firms often face delays and costs when raising fresh funds, an issue that has spurred the creation of an ad-hoc committee within the U.S. Securities and Exchange Commission (SEC).[1] Second, their stocks are relatively illiquid as they are characterized by non-negligible bid-ask spreads, low trading volume, and other microstructure frictions (see, e.g., Hou, Kim, and Werner, 2016; Novy-Marx and Velikov, 2016; Chordia, Roll, and Subrahmanyam, 2011).

This paper develops a dynamic model that studies the relation and the real effects of these two attributes. The model focuses on a small firm that has assets in place—which generate a stochastic flow of revenues—and the opportunity to increase the size of its operations by investing in a growth option. The small firm realistically faces uncertainty in its ability to raise external financing, as in Hugonnier, Malamud, and Morellec (2015). The key departure from previous dynamic corporate finance models with financial constraints is the explicit consideration of the frictions faced by shareholders and liquidity providers when trading the firm stocks, which are shown to importantly affect corporate financial and investment decisions.

To disentangle the forces at play, I start by examining an environment in which shareholders face a constant bid-ask spread when trading the firm stock. To compensate for the ensuing trading losses, investors require a larger return to invest in the stock, similar to Amihud and Mendelson (1986). As a result, the cost of external equity increases and, all else equal, should incentivize the firm to keep more cash. At the same time, however, the greater return required by the investors also generates an offsetting strength: it expands the wedge between the firm's cost of capital and the return on cash, then in-

[1] Small firms are typically less known and more vulnerable to capital market imperfections. In contrast, large, established firms are more likely to have continued relations with financial institution and are less subject to asymmetric information. The U.S. SEC Advisory Committee in Small and Emerging Companies pointed out that small firms often struggle to attract capital, see https://www.sec.gov/spotlight/advisory-committee-on-small-and-emerging-companies.shtml.

creasing the opportunity cost of cash. On net, I show that firms whose stocks are traded at larger bid-ask spreads are more financially constrained. As a result, these firms are more likely to face forced liquidations, as they access external financing less often and keep smaller cash reserves. Moreover, these firms face a severe underinvestment problem, as the additional return required by the investors erodes the profitability of investment opportunities. Overall, firm value decreases.

I show that these effects are robust to a battery of alternative assumptions. First, I investigate the effect of modeling financing frictions as issuance costs (instead of financing uncertainty). I show that bid-ask spreads have a similar impact on cash management, investment, and firm value, irrespective of the way financing frictions are modeled. I also extend the model to allow the firm to borrow from bank credit lines as an additional source of liquidity, and confirm the robustness of the results. Furthermore, I model dynamic investment as in the neoclassical q framework of Bolton, Chen, and Wang (2011), then allowing for cash and capital accumulation. This setup shows that if financial constraints are sufficiently lax so that the firm engages in positive investment, the investment rate decreases with the magnitude of the bid-ask spread—i.e., the bid-ask spread engenders underinvestment.

Next, I relax the assumption that the bid-ask spread is constant and exogenous. To do so, I put more structure into the model and realistically assume that liquidity providers are perfectly competitive and face participation frictions (i.e., participation fees and funding constraints). In this environment, the competitive bid-ask spread not only affects, but also reflects corporate policies. Larger frictions faced by liquidity providers lead, all else equal, to larger bid-ask spreads, consistent with the empirical evidence in Comerton-Forde et al. (2010) and Aragon and Strahan (2012), among others. As shown in the first part of the model, larger bid-ask spreads reduce firm value. When liquidity provision is endogenous, such a decrease in firm value feeds back into the bid-ask spread. Specifically, liquidity providers need to extract larger rents from shocked shareholders as a proportion of the value of their claim to cover their participation fee. As a result, the bid-ask spread widens. Thus, frictions faced by liquidity providers are passed on to the small firm's investors and, through this channel, affect corporate outcomes—exacerbating financial constraints, triggering underinvestment, and increasing the probability of forced liquidations.

The model delivers a rich set of testable predictions. First, it characterizes corporate policies and outcomes for those firms, such as small and micro ones, whose stocks are relatively illiquid because of larger bid-ask spreads and other microstructure frictions. The model predicts that these firms should face severe financial constraints because of their larger costs of external *and* internal equity, which lead them to raise outside funds less often and keep smaller cash reserves. As a result, these firms face higher liquidation risk. Notwithstanding these constraints, these firms should display larger payout rates in the cross-section to compensate for investors' trading frictions, which is consistent with the evidence in Banerjee, Gatchev, and Spindt (2007). These firms should also face a severe underinvestment problem. The quantitative implementation of the model shows that these results are important in magnitude. Moreover, the model can rationalize the evidence that small firms' bid-ask spreads increase when liquidity providers' funding constraints tighten, as illustrated by Anand et al. (2013) and Aragon and Strahan (2012). It also supports the evidence that liquidity supply has important effects on corporate financing and investment (see, e.g. Goldberg, 2020).

Furthermore, the model provides a tractable framework to assess regulatory proposals targeting equity markets from a corporate perspective. Academics and policy-makers have been questioning whether market forces guarantee enough liquidity provision, especially for small-capitalization stocks. Several stock markets have then contemplated the possibility for firms to engage a designated market maker (DMM) to maintain the bid-ask spreads below an agreed-upon level. DMM contracts can help decrease the firm's cost of capital by capping investors' trading costs, but bind the firm to correspond a fee to the DMM. On net, the model suggests that small, financially constrained firms would hardly find this contractual agreement value-enhancing. The model also helps understand the real effects of financial transaction taxes (FTTs). By increasing trading costs for investors, FTTs exacerbate firms' financial constraints, fuel liquidation risk, depress investment, and lower valuations. FTTs would be particularly harmful for small stocks, as these firms already suffer from larger bid-ask spreads.[2]

Related literature Whereas there are many studies focusing on startups or private growth firms in tech industries, there are very few works that explicitly focus on small public firms (see, e.g., Mehran and Peristiani, 2010). Yet, small firms represent a staggering fraction of U.S. listed firms. This paper studies the real, intertwined effects of two market imperfections that grip these firms: (i) frictions faced by the firm when raising fresh funds, and (ii) frictions faced by their shareholders when trading the stock. By showing that the interaction of these frictions increases the firm's liquidation probability, this paper provides a rationale as to why small firms are fading from exchanges, see Kahle and Stulz (2017) and Doidge, Karolyi, and Stulz (2017).

A growing literature shows that frictions affecting stock trading impact corporate policies and outcomes. Fang, Noe, and Tice (2009) show that firms with liquid stocks are more valuable. Campello, Ribas, and Wang (2014) find that stock liquidity improves corporate investment and value. Bakke, Jens, and Whited (2012) show that delisting of a firm's stock (and the ensuing drop in liquidity) leads to a decrease in employment and, to a lower extent, investment and savings. Nyborg and Wang (2019) show that stock liquidity increases a firm's propensity to hold cash. Banerjee, Gatchev, and Spindt (2007) reveal that firms with illiquid stocks pay out more dividends. Brogaard, Li, and Xia (2017) find that stock liquidity reduces firms' bankruptcy risk. This paper develops a unified framework that provides theoretical grounds to these empirical findings.

Since the 2007–2009 financial crisis, there has been a large interest in understanding the effects of liquidity providers' funding liquidity on market liquidity. Many empirical works show that frictions faced by intermediaries affect the liquidity of their traded stocks; see, for instance, Anand et al. (2013); Hameed, Kang, and Viswanathan (2010); Aragon and Strahan (2012); Comerton-Forde et al. (2010). In the meanwhile, a theoreti-

[2] The model then suggests that policies making the FTT contingent on a firm's market capitalization could relieve these harmful effects. That said, the analysis is silent on the desirability (the welfare gains) arising from this tax, in that the focus of the paper is on the effects on the corporate sector.

cal literature has developed to understand the dynamics of liquidity demand and supply and their impact on asset prices; see, for instance, Budish, Cramton, and Shim (2015) or Huang and Wang (2010, 2009). These models abstract away from informational frictions but, rather, focus on financial constraints and margin requirements faced by market participants (like intermediaries, broker-dealers, and other liquidity providers). A key assumption in these models is that the security traded by market participants promises a constant (exogenous) flow of dividends. In contrast, the current paper endogenizes the dividend flow associated with the traded security by investigating the optimal corporate policies of the issuing firm.

The influential work by Brunnermeier and Petersen (2008) shows that there is a two-ways link between an asset's market liquidity and traders' funding liquidity. Traders provide market liquidity, which in turn depends on their funding ability. Because of margin requirements, traders' funding depends on the assets' market liquidity. The current paper instead focuses on the relation between the funding liquidity of a given (nonfinancial) small/micro firm and the market liquidity of its stocks. It shows that firm financing frictions and stock microstructure frictions reinforce each other, then providing a rationale as to why the degree of financial constraints and stock market illiquidity are disproportionately greater for small/micro stocks vis-à-vis large firms.

Finally, this paper contributes to the strand of dynamic corporate finance models with financing frictions, including Décamps et al. (2011, henceforth DMRV); Bolton, Chen, and Wang (2011, henceforth BCW); Hugonnier, Malamud, and Morellec (2015, henceforth HMM); Malamud and Zucchi (2019); or Della Seta, Morellec, and Zucchi (Forthcoming). These papers show that financing frictions, such as costs or uncertainty in raising external funds, should increase a firm's propensity to keep precautionary reserves. While these extant papers impose an exogenous cost of holding cash, the current model shows that this cost can arise endogenously when accounting for trading frictions faced by firm shareholders. Notably, it illustrates that trading frictions impact both the cost of internal and external financing, then affecting corporate policies and value.

The paper proceeds as follows. Section 2 describes the model. Section 3 analyzes the effects of an exogenous bid-ask spread on corporate policies, and assesses the robustness of the results to alternative modeling assumptions. Section 4 endogenizes the bid-ask spread and studies some regulatory proposals targeting financial equity markets from a corporate perspective. Section 5 concludes. Proofs are gathered in the Appendix.

2 The model

Time is continuous, and uncertainty is modeled by a probability space $(\Omega, \mathcal{F}, P)$ equipped with a filtration $(\mathcal{F}_t)_{t\geq 0}$. Agents are risk-neutral and discount cash flows at a constant rate $\rho > 0$.

The firm I consider a small firm operating a set of assets in place, which generate a continuous and stochastic flow of revenues. The flow of revenues is modeled as an arithmetic Brownian motion, $(Y_t)_{t\geq 0}$, whose dynamics evolve as

$$dY_t = \mu dt + \sigma dZ_t. \tag{1}$$

The parameters μ and σ are strictly positive and represent the mean and volatility of corporate revenues, and $(Z_t)_{t\geq 0}$ is a standard Brownian motion. The firm has access to a growth option that has the potential to increase its income stream from dY_t to $dY_t^+ = dY_t + (\mu_+ - \mu)dt$, $\mu_+ > \mu$, by paying a lump-sum cost $I > 0$. That is, the cash flow drift can assume two values $\mu_i = \{\mu, \mu_+\}$. Investment is assumed to be irreversible.

The cash flow process in equation (1) implies that the firm can make operating profits and losses. If capital supply was perfectly elastic, operating shortfalls could be covered by raising outside financing immediately and at no cost. In practice, small firms face financing frictions, such as uncertainty or costs in their ability to raise funds. I model capital supply uncertainty as in HMM and assume that the firm raises new funds at the jump times of a Poisson process, $(N_t)_{t\geq 0}$, with intensity λ. That is, if the firm decides to raise outside funds, the expected financing lag is $1/\lambda$ periods. When $\lambda \to 0$, the firm cannot raise external funds at all (equivalently, it takes an infinite waiting period to raise fresh funds upon searching) and relies on cash reserves to cover unexpected operating losses. When $\lambda \to \infty$, conversely, the waiting time upon searching for external funds is zero—i.e., the firm has access to outside financing whenever needed at no delays. Notably, as shown in the paper, the discount on newly-issued equity is related to trading frictions faced by firms' investors. In Section 3.3.1, I show that the specific assumptions regarding the firm's financing frictions are without loss of generality—the main results of the paper continue to hold if financing frictions are modeled as issuance costs, as in DMRV or BCW.

Because capital supply is uncertain, the firm has incentives to retain earnings in cash reserves. I denote by $(C_t)_{t\geq 0}$ the firm's cash reserves at any $t \geq 0$. Cash reserves earn a constant rate, $r \leq \rho$. Whenever $r < \rho$, keeping cash entails an opportunity cost.[3] In contrast with extant cash holdings models—in which the strict inequality $r < \rho$ is needed to depart from the corner solution featuring firms piling infinite cash reserves—I allow for the $r = \rho$ case. The cash reserves process satisfies:[4]

$$dC_t = rC_t dt + \mu_i dt + \sigma dZ_t - dD_t + f_t dN_t. \tag{2}$$

In this equation, $dD_t \geq 0$ represents the instantaneous flow of payouts at time t. The process $(D_t)_{t\geq 0}$ is non-decreasing, reflecting shareholders' limited liability. $f_t \geq 0$ denotes the instantaneous inflow of funds when financing opportunities arise, in which case management stores the proceeds in the cash reserves. This assumption is consistent with the strong, positive correlation between equity issues and cash accumulation documented by McLean (2011) or Eisfeldt and Muir (2016). Notably, D and f are endogenous. Equation (2) implies that the firm's cash reserves increase with external financing, retained earnings, and the interest earned on cash, whereas they decrease with payouts and operating

[3]This cost can be interpreted as a free cash flow problem à la Jensen (1986) or as tax disadvantages (see Graham, 2000).

[4]When investment occurs (meaning that the cash flow drift goes from μ to μ_+), the cost I is financed either with cash or external financing. Because the paper focuses on the decision of whether or not to invest (rather than on the investment timing), I do not explicitly model the outflow I when the growth option is exercised (which could be financed with outside funds).

losses. The level of the firm's cash reserves is a gauge of the firm's financial strength.

Management can distribute cash and liquidate the firm's assets at any time. Yet, liquidation is inefficient, as the recovery value of assets, denoted by ℓ, is smaller than the firm's first best, μ_i/ρ. These costs erode a fraction, $1 - \phi \in (0, 1]$, of the firm's first best, so the liquidation value is $\ell = \phi\mu_i/\rho$. I denote by τ the endogenous time of liquidation.

Transacting the firm stocks The key departure from previous dynamic corporate finance models with financing frictions is the explicit consideration of investors' stock transactions and the costs or losses thereof. There are two types of risk-neutral traders: investors (who may buy, hold, and eventually sell the stock) and trading firms (or liquidity providers, which ease investors' trading).

Investors are ex-ante identical and infinitely lived. Each of them has measure zero and cannot short sell. Investors can be hit by liquidity shocks. Following previous contributions (e.g., Bessembinder, Hao, and Zheng, 2015), liquidity shocks lead to a sudden need for liquidity that reduces the subjective valuation of the asset by a fraction χ.[5] Thus, χ can be interpreted as the opportunity cost of being locked into an undesired asset position—for instance, because of take-it-or-leave-it investment opportunities, unpredictable financing needs, or unpredictable changes in hedging needs. The liquidity shock vanishes once the shocked investor either sells his stock or he bears the loss χ. Liquidity shocks are independent across investors and occur at the jump times of a Poisson process with intensity $\delta > 0$. In turn, non-liquidity-shocked shareholders have no immediate need to trade and, thus, are indifferent between keeping the stock or selling it at its fundamental value.[6]

Trading firms are agents who maintain an active presence and provide liquidity in the market of the stock.[7] They have no intrinsic demand to buy or sell the firm's assets and are

[5]As in Bessembinder, Hao, and Zheng (2015), the cost of liquidity shocks is proportional to the fundamental value of the asset held by the investor.

[6]Following previous contributions (see, e.g., He and Milbradt (2014)), I assume that the mass of non-liquidity-shocked investors is larger than that of liquidity-shocked shareholders, without loss of generality.

[7]Trading firms can be interpreted as high frequency traders, market makers, or algorithmic traders as in Budish, Cramton, and Shim (2015), or simply as agents maintaining a constance market presence

not subject to liquidity shocks. Trading firms compete à la Bertrand in providing liquidity to shocked investors. Trading firms pay a fixed flow cost as long as they are active in the market of the stock, denoted by γ, which can be interpreted as the cost of monitoring and processing market movements. Moreover, I assume that liquidity providers are financially constrained. Their cost of funding erodes a fraction κ of their gross gain from liquidity provision.

Trading firms post bid and ask quotes. On the ask side, they trade with non-liquidity-shocked investors, who are indifferent between staying out of the market or buying the stock at its fundamental value (i.e., they do not have an immediate need to trade). As a result, the gain to trading firms on this side of the transaction is null. On the bid side, trading firms transact with shocked shareholders. Because shocked shareholders value the asset at a discount χ, trading firms can extract surplus from this side of the transaction. The trading firms' gain from transacting with shocked shareholders is denoted by $\eta \leq \chi$. The quantity η then represents the difference between the ask and the bid price (i.e., the bid-ask spread). To disentangle the forces at play, this quantity is taken as exogenous in Section 3, and then derived endogenously in Section 4.

Equilibrium Corporate decisions—cash retention and payout (D), financing (f), liquidation (τ), and investment (I)—are set to maximize:

$$V(c) = \sup_{(D,f,\tau,I)} \mathbb{E}\left[\int_0^\tau e^{-\rho t}\left(dD_t - f_t\, dN_t - dB_t\right) + e^{-\rho\tau}\ell\right], \tag{3}$$

which represents the expected present value of payouts net of outside financing (the first term) plus the liquidation value of assets (the second term). The expected flow of payouts to shareholders is drained by trading losses (whose cumulative process is denoted by B_t). Payouts and the bid-ask spread are endogenously determined. In equilibrium, the bid-ask spread leaves trading firms indifferent between providing liquidity in the market of the stock and staying out of the market. Via the trading firms' decision to provide liquidity,

such as trading desks and hedge funds, as in Huang and Wang (2010). For simplicity, the size of the trading firm sector is normalized to one.

the bid-ask spread not only affects, but also reflects corporate policies and value.

2.1 Discussion of the assumptions

The model nests trading frictions faced by firm's shareholders into a dynamic corporate finance model with financial constraints. As such, the model is especially relevant for small and micro firms, which suffer from severe financial constraints and whose stocks are relatively illiquid and traded at non-negligible bid-ask spreads.[8] To keep the analysis tractable, the paper takes a parsimonious approach in modeling these frictions.

Specifically, the modeling of secondary market transactions is flexible enough to be applicable to stocks traded on major exchanges as well as to stocks traded in over-the-counter markets. One key difference of this paper vis-à-vis models of the effects of liquidity demand/supply on asset valuations is the explicit focus on the policies of the issuing firm (rather than on a full-fledged investigation of the dynamics of secondary market transactions). Notably, whereas previous contributions usually take the flow of dividends associated with a given stock as constant, the current paper endogenizes it. To keep the analysis simple, two assumptions are made. First, the trading costs borne by shocked (selling) investors are positive, whereas the costs borne by (buying) non-shocked investors are zero. This assumption is consistent with Brennan et al. (2012), who show that sell-order frictions are priced more strongly than buy-order ones.[9] Second, as in Budish, Cramton, and Shim (2015) or Huang and Wang (2010), the model abstracts from asymmetric information about the current value of the firm. Indeed, recent empirical evidence shows the importance of the non-information component of trading costs on asset prices (e.g., Chung and Huh, 2016). This paper builds on this strand and seeks to focus on the implications for corporate outcomes.

[8]Large firms have an easier access to several sources of financing (like corporate bonds or commercial paper), and their stocks are traded at tiny bid-ask spreads. Whereas large firms dwarf small firms capitalization-wise, the number of small and micro firms exceeds by far that of large firms, representing more than 80% of the number of listed firms.

[9]Brennan et al. (2012) show that the pricing of illiquidity emanates principally from the sell side. The underlying idea is that agents seldom face needs to buy stock urgently, but unexpected needs for cash may force them to suddenly sell stocks.

Turning to the firm's financing, the paper closely follows HMM and assumes that the firm faces uncertainty in its ability to raise fresh funds, an issue that is especially severe for small firms, as also pointed out by the U.S. SEC Advisory Committee in Small and Emerging Companies. As illustrated by the survey evidence in Lins, Servaes, and Tufano (2010), financing uncertainty is one of the top reasons behind corporate cash stockpiling. Thus, the model realistically allows firm management to accumulate earnings in cash reserves to withstand operating losses. To assess the robustness of our framework to alternative modeling of the firm's financing frictions, Section 3.3.1 assumes that the firm faces issuance costs whenever raising new equity, as in DMRV and BCW. This extension confirms the qualitative and quantitative robustness of our results.

It is also worth noting that small/micro firms find it too costly (or unfeasible) to access bond financing. Rather, these firms usually access debt by borrowing from banks—e.g., by securing themselves credit line availability. While the baseline version of the paper abstracts from this source of financing, credit line availability is introduced in Section 3.3.3. The model predictions are shown to be robust to this extension.

Finally, whereas the baseline model assumes that the firm can expand the size of its operations through the exercise of a lumpy growth option, Section 3.3.2 considers continuous investment subject to adjustment costs, as BCW. This model extension allows to understand how trading frictions faced by firm investors (such as bid-ask spreads) affect the accumulation of capital by affecting the firm's investment/disinvestment rate, then strengthening and expanding the results of the baseline model.

3 The effect of bid-ask spreads on corporate policies

To disentangle the economic effects at play, it is useful to start by analyzing the firm's decision-making problem when shareholders face an exogenous bid-ask spread η. This assumption will be relaxed in Section 4.

3.1 Deriving firm value

Because liquidity shocks are independent across investors, a measure δdt of shareholders is shocked on each time interval. Shareholders seek to sell the stock as soon as hit by a liquidity shock. When trading with shocked shareholders, trading firms capture a fraction η of the surplus created, which means that the transaction price of the aggregate claim of shocked shareholders is: $\delta\Phi(c) \equiv (1-\eta)\delta V(c)$. As long as $\delta\Phi(c) > (1-\chi)\delta V(c)$ (which is the case if $\eta < \chi$), liquidity-shocked shareholders are better off selling the stock (which entails the loss η) than keeping it (which entails the loss χ). The quantity

$$\delta[\Phi(c) - V(c)] = -\delta\eta V(c)$$

represents the associated (aggregate) loss to shocked shareholders.

I next show that this loss affects corporate policies and firm value. Assume first that the firm does not have any growth options[10] and consider the optimal cash retention and payout policies. As in previous cash management models, the benefit of holding cash decreases with cash reserves. Its (opportunity) cost is the wedge between the return required by the investors and the return on cash. I conjecture (and verify) that there is a target cash level, C_V, at which the cost and benefit of cash are equalized. Above C_V, it is optimal to pay excess cash out. Below C_V, shareholders retain earnings in cash reserves and search for financing. When operating losses cannot be covered by drawing funds from cash reserves or by raising fresh equity, the firm is forced into liquidation. Liquidation then occurs the first time that the cash reserves process hits zero:

$$\tau = \inf\{t \geq 0 : C_t \leq 0\}. \quad (4)$$

By Itô's lemma, equity value satisfies the following ordinary differential equation

[10]Solving for firm value when there are no growth option is auxiliary to studying the optimal investment rule, as in HMM. The optimal investment rule is reported in Proposition 4.

(ODE) for any $c < C_V$:

$$\rho V(c) = (rc + \mu) V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda \sup_f [V(c+f) - f - V(c)] + \delta [\Phi(c) - V(c)]. \quad (5)$$

The left-hand side is the return required by the investors. The first two terms on the right-hand side represent the effect of cash retention and cash flow volatility on equity value. The third term represents the surplus from raising external financing, i.e., the probability-weighted surplus accruing to incumbent shareholders. In the appendix, $V(c) - c$ is shown to increase with c, so it is optimal to raise the cash buffer up to C_V whenever financing opportunities arise.[11] Thus, the optimal refinancing amount is $f(c, \Phi) = C_V(\Phi) - c$. The last term reflects the loss borne by liquidity-shocked investors when selling the firm's stocks. Substituting $\Phi(c)$ and $f(c, \Phi)$ into equation (5) gives

$$(\rho + \delta\eta) V(c) = (rc + \mu) V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda [V(C_V) - V(c) - C_V + c]. \quad (6)$$

The left-hand side reveals that the bid-ask spread increases the return required by investors by $\delta\eta \geq 0$. This additional compensation leads to an increase in the opportunity cost of cash from $\rho - r$ to $\rho + \delta\eta - r$. Equation (6) is solved subject to the following boundary conditions. The firm is liquidated when cash is exhausted and the firm cannot raise new funds. Thus, the condition

$$V(0) = \ell \quad (7)$$

holds when the firm runs out of cash. Moreover, it is optimal to distribute all the cash exceeding C_V as payouts.[12] Firm value is thus linear above C_V: $V(c) = V(C_V) + c - C_V$ for any $c \geq C_V$. Subtracting $V(c)$ from both sides of this equation, dividing by $c - C_V$,

[11] The marginal value of cash satisfies $V'(c) \geq 1$ (see Lemma 6 in Appendix A for a proof). This implies that the first derivative of $V(c) - c$ is non-negative.

[12] As shown in the following, it is never optimal to buy back the shares of shocked investors when $c < C_V$.

and taking the limit as c tends to C_V gives

$$\lim_{c \uparrow C_V} V'(c) = 1. \tag{8}$$

That is, it is optimal to start paying out cash when the marginal value of one dollar inside the firm equals the value of a dollar paid out to shareholders. The target cash level that maximizes shareholder value is determined by the super-contact condition,

$$\lim_{c \uparrow C_V} V''(c) = 0. \tag{9}$$

Both the target cash level C_V and the issue size f depend on η. Below I further investigate corporate policies vis-à-vis an environment in which the bid-ask spread is zero.

3.2 Analyzing corporate policies

Cash management and payouts In the presence of financing frictions, the benefit of cash stems from guaranteeing financial flexibility to the firm. The firm's target cash level balances this benefit against the opportunity cost of holding cash. A key contribution of this paper is to show that the bid-ask spread associated with a firm's stock affects this tradeoff by impacting both the cost and benefit of cash. By increasing the return required by the investors, a positive bid-ask spread leads to: (i) an increase in the cost of equity, and (ii) an increase in the cost of cash, as it expands the wedge between the return on cash and the firm's cost of capital.[13] In this section, I investigate the net effect of these strengths on the incentives to keep cash.

Consider first the extreme case in which financing frictions are so severe that the firm has no access to external financing (i.e., $\lambda = 0$). The bid-ask spread does not affect firm's refinancing decisions in this case (as the firm has no access to outside financing) and, thus, does not affect the benefit of cash directly. However, bid-ask spreads increase the

[13] This model then delivers finite target cash levels even when r and ρ coincide. In previous dynamic cash management models, holding cash is not costly if r and ρ coincide and, thus, a financially constrained firm would pile infinite cash reserves if $r = \rho$ holds.

cost of holding cash. As a result, the bid-ask spread leads to a decrease in the target cash level.

When the firm has access to external financing ($\lambda > 0$), the bid-ask spread has opposing effects on the incentive to keep cash. First, the larger opportunity cost of cash leads the firm to reduce its target cash level. Second, the larger cost of equity reduces the surplus from outside financing accruing to existing shareholders. All else equal, this effect makes cash reserves more valuable and, thus, generates an incentive to increase the target cash level. The next proposition shows that the first effect dominates. I denote by C^* the target cash level when the bid-ask spread is zero.

Proposition 1 (Cash management) *On net, a positive bid-ask spread leads to a decrease in the target cash level, all else equal; i.e., the inequality $C_V \leq C^*$ holds. The greater the bid-ask spread η, the smaller the target level C_V.*

Proposition 1 then illustrates that a positive bid-ask spread leads firms to decrease their optimal cash reserves (see Appendix A.1 for a proof). This prediction is consistent with Nyborg and Wang (2019), who find that stock liquidity (using the relative effective bid-ask spread as a measure, among others) increases a firm's propensity to hold cash.

Providing liquidity to investors via payouts Cash retention and payout decisions are closely intertwined. I define the payout probability as follows:

$$P^p(c, C_V) = E_c\left[e^{-\lambda\tau_d(C_V)}\right],$$

where $\tau_d(C_V)$ represents the first time that the cash reserves process, initially at c, reaches the payout threshold C_V. The bid-ask spread enters this probability through $\tau_d(C_V)$—by affecting the target cash level, the bid-ask spread affects the firm's payout frequency. The following proposition studies this probability compared to the case in which the bid-ask spread is zero (in which case the payout probability is denoted by $P^p(c, C^*)$).

Proposition 2 (Payout probability) *A positive bid-ask spread leads to an increase in the firm's payout probability, i.e., $P^p(c, C_V(\eta)) > P^p(c, C^*)$.*

Proposition 2 implies that a firm pays out more dividends if its stock is traded at a larger bid-ask spread. In so doing, the firm compensates shareholders for the frictions they bear when trading the stock. This finding is in line with Banerjee, Gatchev, and Spindt (2007), who suggest that investors view stock market liquidity and dividends as substitutes. When a firm's bid-ask spread is small, investors can create dividends to themselves by cashing out their investment. When the bid-ask spread is large, investors require the firm to pay out more dividends.

A question then arises as to whether the firm should commit to a payout flow to investors (for instance, by repurchasing the shares of shocked investors) for any $c < C_V$, then effectively serving as a liquidity provider. In so doing, the firm would then decrease its cost of capital. I next show that this policy would be suboptimal. To this end, suppose that the firm follows this policy and repurchases the shares of shocked investors at their fair value, meaning that the firm bears a constant cash outflow equal to $\delta V(c)$ on any time interval. Firm value would satisfy

$$\rho V(c) = \left[rc + \mu - \delta V(c)\right] V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda \left[V(C_V) - C_V + c - V(c)\right] , \qquad (10)$$

subject to the same boundary conditions in Section 3.1. Equation (10) differs from (6) as there is no loss borne by shocked investors, thanks to the firm's commitment to buy back their shares. Yet, this policy is suboptimal, as the decrease in firm value due to the flow cost of repurchasing shares of shocked shareholders (i.e., the term $\delta V(c)V'(c)$ on the right-hand side of this equation) is larger than the decrease in firm value due to the higher return required by investors (i.e., the term $\delta V(c)\eta$ in equation (6)). The reason is that, for any $c < C_V$, the marginal value of cash is greater inside the firm than if paid out ($V'(c) \geq 1 > \eta > 0$; see Lemma 6 in Appendix A).

Alternatively, the firm could buy back the shares of shocked shareholders at a price smaller than $\delta V(c)$. Yet, this price cannot be smaller than $\delta V(c)(1 - \eta)$—otherwise shocked shareholders would be better off selling the stock to trading firms than to the firm. That is, the firm needs to buy back shares at a price at least equal to $\delta V(c)(1 - \eta)$.

When following this policy, firm value would satisfy

$$(\rho + \delta\eta)V(c) = [rc + \mu - (1-\eta)\delta V(c)]\, V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda\,[V(C_V) - C_V + c - V(c)]\,.$$

Again, this policy decreases firm value compared to that derived in Section 3.1. In fact, the firm would need to bear the greater return required by the investors as well as to commit to repurchase shares for $c < C_V$, then draining the firm's cash flows. It is then possible to generalize that repurchasing shares is suboptimal for any $c < C_V$ and at any price in the interval $[\delta V(c)(1-\eta), \delta V(c)]$.

This analysis suggests that payouts (share repurchases or dividends) are suboptimal when cash reserves are below the target level C_V. The reason is that, for any $c < C_V$, cash is more valuable inside the firm ($V'(c) > 1$) than if paid out. Instead of repurchasing shares for any level of cash reserves c, the optimal policy prescribes that the firm should "provide liquidity" to shareholders by reducing the target cash level with respect to the benchmark case with zero bid-ask spread (as from Proposition 1), meaning that the payout threshold is hit more often (as shown by Proposition 2).

Liquidation and financing Because the bid-ask spread increases the cost of internal and external equity, the firm's financial resilience should also be affected. To investigate this important aspect, I study the firm's probability of liquidation and financing. I define the probability of liquidation while the firm is searching for external funds as

$$P^l(c, C_V(\eta)) = \ E_c\left[e^{-\lambda\tau(C_V)}\right]$$

and, complementarily, the probability of external financing is defined as $P^f(c, C_V(\eta)) = E_c\left[1 - e^{-\lambda\tau(C_V)}\right]$. The bid-ask spread enters these probabilities through $\tau(C_V)$, representing the first time that the cash process, reflected from above at C_V, is absorbed at zero. The following proposition studies these probabilities vis-a-vis the case in which the bid-ask spread is zero.

Proposition 3 (Probability of liquidation and financing) *A positive bid-ask spread increases the firm's probability of liquidation, $P^l(c, C_V(\eta)) > P^l(c, C^*)$, and decreases the probability of external financing, $P^f(c, C_V(\eta)) < P^f(c, C^*)$.*

Taken together, these results suggest that bid-ask spreads exacerbate firms' financial constraints through different channels. First, they increase the cost of internal financing, making the firm less willing to keep cash reserves. Second, they reduce the firm's probability of issuing fresh equity by reducing the surplus accruing to current shareholders. As a result, a positive bid-ask spread increase the probability of firm's liquidation, consistent with Brogaard, Li, and Xia (2017).

Investment decisions The analysis so far focuses on the case in which the firm has no growth opportunities (i.e., it conditions on a given drift μ_i). Following Décamps and Villeneuve (2007) and HMM, the value of the firm with no growth opportunities serves to derive the zero-NPV cost—i.e., the maximum amount that the firm is willing to pay to increase the cash flow drift from μ to μ_+. The next proposition illustrates how the bid-ask spread affects such zero-NPV cost (see Appendix A.4 for a proof).

Proposition 4 *A positive bid-ask spread reduces the maximum amount that the firm is willing to pay to invest in the growth option. Specifically, the zero-NPV cost is given by*

$$I_V = \frac{\mu_+ - \mu}{\rho + \delta\eta} - (C_{V+} - C_V)\left[1 - \frac{r}{\rho + \delta\eta}\right], \tag{11}$$

where C_{V+} denotes the target cash level after the growth option is exercised.

If the bid-ask spread was zero, the zero-NPV cost would be

$$I^* = \frac{\mu_+ - \mu}{\rho} - (C^*_+ - C^*)\left(1 - \frac{r}{\rho}\right), \tag{12}$$

with C^*_+ denoting the post-investment target cash level. Comparing (11) and (12) reveals that a positive bid-ask spread leads to a decrease in the investment reservation price (i.e.,

$I_V < I^*$)—that is, it reduces the maximum amount that the firm is willing to pay to exercise the growth option. If the investment cost lies in the interval $[I_V, I^*]$, the growth option has negative NPV if the bid-ask spread is positive ($\eta > 0$), whereas it has positive NPV if the bid-ask spread is zero. The gap between the zero-NPV costs I_V and I^* can be approximated as follows:[14]

$$\Delta I_V = I_V - I^* \approx -\frac{\mu^+ - \mu}{\rho} \frac{\delta\eta}{\rho + \delta\eta} < 0. \tag{13}$$

Notably, the gap between I_V and I^* increases with η. That is, the underinvestment problem worsens if the bid-ask spread widens, a result that is empirically consistent with Campello, Ribas, and Wang (2014) and Amihud and Levi (2019). Section 3.3.2 confirms and extends this prediction to the case in which the firm optimally adjusts its investment rate continuously, as in the neoclassical framework of Bolton, Chen, and Wang (2011).

Quantitative analysis This section provides a quantitative assessment of the effect of bid-ask spreads on corporate policies. Table 1 reports the baseline parameterization. The risk-free rate ρ is set to 2%, and the return on cash is set to 1%. The resulting opportunity cost of cash is equal to 1%, as in BCW and DMRV. Because small firms tend to have lower profitability in the cross section (see, e.g., Fama and French, 2008), the cash flow drift $\mu = 0.05$ is set to be lower than the value used by DMRV and consistent with the bottom range of values in Whited and Wu (2006). Upon exercising the growth option, the cash flow drift is assumed to be 20% bigger (i.e., $\mu_+ = 0.06$). I set $\sigma = 0.12$, which is consistent with Graham, Leary, and Roberts (2015) and is higher than the value set by DMRV, as small firms have more volatile cash flows. Furthermore, I base liquidation costs on the estimates of Glover (2016) and set $\phi = 0.55$. The parameter λ is set to 0.75, which is consistent with the frequency of equity issues by small firms reported by Fama and French (2005). The intensity of the liquidity shock is set to $\delta = 0.7$, as in He and Milbradt (2014). The magnitude of the bid-ask spread is varied extensively throughout

[14] The difference between target cash levels (the second term in the expressions of I_V and I^*) plays a second-order effect, as in the model of HMM.

the analysis.[15]

Table 2 illustrates the impact of bid-ask spreads of various magnitude on the target cash level, the probability of external financing, the probability of liquidation, the probability of payout, the investment reservation price (i.e., the zero-NPV cost), and firm value. For a bid-ask spread equal to 60 basis points, the target cash level decreases by more than 13% with respect to the case in which the bid-ask spread is zero. Because the bid-ask spread engenders a wedge between the return required by the investors and the return on cash, the firm pays out cash to investors more often. In fact, the table shows that the probability of payout increases on average by about 3.1% when the bid-ask is equal to 60 basis points.[16] Moreover, the maximum investment threshold decreases by almost 18%, engendering a notable underinvestment problem. Because both external and internal financing are more expensive if the bid-ask spread is positive, the probability of liquidation increases by 2%, on average. Notably, the increase in the probability of liquidation is greater as cash reserves approach depletion. Specifically, Table 3 illustrates the impact of bid-ask spreads on the probability of liquidation, at different cash levels. For instance, when the firm stands at $C_V/4$, the probability of liquidation is equal to 14.1% if the bid-ask spread is zero, and is equal to 18.3% if the bid-ask spread is equal to 60 basis points. It is also worth noting that, for a given cumulative shock, liquidation becomes relatively more likely if the bid-ask spread is larger. If the bid-ask spread is zero, a series of shocks reducing the cash buffer from $C^*/2$ to $C^*/4$ increases the probability of liquidation from 1.97% to 14.1%. If the bid-ask spread is equal to 60 basis points, the reduction from $C_V/2$ to $C_V/4$ is caused by a cumulative loss about 13% smaller (than in the case in which the bid-ask spread is zero), but it increases the probability of liquidation

[15] Chung and Zhang (2014) report the median bid-ask spread (calculated using TAQ data) for firms sorted by quintiles of market capitalization over the period 1993-2009. They report that the median bid-ask spread of smaller quintile firms is 0.0195 for NYSE/AMEX stocks and 0.0501 for NASDAQ stocks. They also note that the bid-ask spread has decreased over time (see also Hasbrouck, 2009): The median bid-ask spread for (all capitalization) NYSE/AMEX stocks went from 0.0094 in 1993 to 0.0034 in 2009, and from 0.0346 in 1993 to 0.0067 in 2009. In the model parameterization, I take a conservative approach and take a relatively low value for the bid-ask spread. In so doing, I show that even small bid-ask spreads can bear substantial impact on corporate policies and value.

[16] To calculate these probabilities, I follow HMM and calculate the average for a cross-section of firms with cash reserves uniformly distributed between 0 and C_V.

from 3.35% to 18.3%. Overall, a bid-ask spread equal to 60 basis points leads to an 18% reduction in firm value. Table 2 shows that such a loss is sizable even for smaller bid-ask spreads.

The analysis then advances the following testable predictions. First, firms whose stocks trade at a larger bid-ask spread are more financially constrained, as they tap the equity market less often and keep less cash. Second, the firm is less resilient to negative operating shocks, and the probability of liquidation increases more steeply after a given cumulative shock. Third, the firm underinvests, as the additional return required by the investors erodes the profitability of investment opportunities and reduces the firm's reservation price. Overall, firm value decreases considerably.

3.3 Robustness to alternative assumptions

3.3.1 Modeling financing frictions as issuance costs

The baseline setup in Section 2 models financing frictions as capital supply uncertainty, as in the dynamic model of HMM and consistent with the difficulties faced by small firms when raising external funds. This section assesses the robustness of this analysis by considering an alternative setup in which financing frictions are modeled as issuance costs, similar to DMRV or BCW. In this setup, the firm can choose the timing of equity issuances (rather than waiting for the arrival of financing opportunities).

Specifically, I assume that external financing entails proportional and fixed costs, which are denoted by ϵ and ω, respectively. These costs prompt the firm to keep precautionary cash reserves—the target cash level is still denoted by C_V. When it is optimal for the firm to save earning in the cash reserves (i.e., for any $c < C_V$), firm value satisfies the following equation:

$$\rho V(c) = (rc + \mu)\, V'(c) + \frac{\sigma^2}{2} V''(c) + \delta\, [\Phi(c) - V(c)]\,, \tag{14}$$

which differs from equation (5) as there is no jump in firm value due to the stochastic arrival of financing opportunities. Rather, the firm sets financing decisions to minimize issuance costs. To economize on the fixed cost, the firm raises funds in a lumpy fashion when cash reserves are depleted. Denote the optimal issue size by C_*. The following condition then holds:

$$V(0) = V(C_*) - (1+\epsilon)C_* - \omega \tag{15}$$

which implies that firm value at $c = 0$ (the left-hand side of this equation) equals the firm's continuation value net of issuance cost (the right-hand side). Notably, it is optimal for the firm to raise external financing if the following inequality

$$V(C_*) - (1+\epsilon)C_* - \omega > \ell \tag{16}$$

holds, which guarantees that the firm's continuation value (the left-hand side) is larger than the liquidation value of the firm (the right-hand side). The optimal issue size C_* satisfies the following condition

$$V'(C_*) = 1 + \epsilon, \tag{17}$$

which warrants that the marginal benefit (the left-hand side of this equation) and cost of external financing (the right-hand side) are equalized at C_*. Lastly, Equation (14) is subject to boundary conditions at the target cash level that are similar to (8) and (9).

Table 4 shows the effect of bid-ask spreads on the target cash level, the optimal issuance size, the zero-NPV investment cost, and firm value. I use the baseline parameters reported in Table 1 and, in addition, consider two sets of values for the issuance cost parameters. In the top panel, I assume that ϵ is equal to 0.06 and ω is equal to 0.01 as in BCW, whereas in the bottom panel I assume that ϵ is equal to 0.10 and ω is equal to 0.03, which account for the heterogeneity in financing costs documented by Hennessy and Whited (2007). Table 4 confirms the results in Section 3.2 that the target cash level decreases with the bid-ask spread. If the bid-ask spread is equal to 60 basis points, the target cash level decreases by about 9% with respect to the case with zero bid-ask spread.

In addition, as financig opportunities are not stochastic, this model extension allows to investigate how bid-ask spreads affect the optimal size of equity issuances (C_*). Table 4 illustrates that the size of equity issuances decreases with the bid-ask spread.[17] For a bid-ask spread equal to 60 basis points, the optimal size of equity issuances decreases by about 4% lower compared to the benchmark case in which the bid-ask spread is zero. Furthermore, Table 4 confirms that the maximum investment cost decreases with the magnitude of the bid-ask spread, and so does firm value. For a bid-ask spread equal to 60 basis points, the zero-NPV cost decreases by about 17% compared to the case in which the bid-ask spread is zero. Also, a bid-ask spread equal to 60 basis points leads to a decrease in firm value equal to about 18%, consistent with the results in Section 3.2 (and across the top and bottom panel). This analysis then confirms that the impact of bid-ask spreads holds irrespective of the way financing frictions are modeled—i.e., being costs or uncertainty in raising new funds.

3.3.2 Continuous investment and capital accumulation

In this section, I relax the assumption that the firm has just one lumpy investment opportunity. Instead, following BCW, I assume that the firm can continuously adjust its capital stock, denoted by K_t. Specifically, I assume that the firm's capital stock evolves as follows:

$$dK_t = (i - d)K_t dt, \tag{18}$$

where i denotes the firm's endogenous investment rate, and d represents the depreciation rate of capital. Following BCW, the price of capital is normalized to one. Moreover, investment in capital entails a quadratic adjustment cost equal to

$$G(i, K) = g(i)K = \frac{\theta}{2} i^2 K. \tag{19}$$

[17] As in previous cash management models in which financing frictions are modeled as issuance costs, the firm finds it optimal to either issue the same amount whenever its cash reserves are depleted, or to liquidate the first time that cash reserves fall to zero. As a result, I do not report the analyis of the probabilities of refinancing and liquidation.

As a result, the dynamics of the firm's operating profit satisfy

$$dX_t = K_t \left[(\mu - i - g(i))dt + \sigma dZ_t \right], \tag{20}$$

i.e., operating profits are proportional to the capital stock. The parameters $\mu > 0$ and $\sigma > 0$ are constants, and Z_t is a standard Brownian motion, similar to equation (1).

In this setup, firm value $V(C, K)$ is a function of cash reserves and of the capital stock. Standard arguments give the HJB equation reported in Appendix A.5. Exploiting homogeneity, the firm's optimization problem reduces to one state variable, being cash reserves normalized by capital $w = C/K$. Consistently, $v(w)$ is defined as firm value scaled by capital K_t. Calculations reported in Appendix A.5 show that the firm's optimal investment rate is given by:

$$i(w) = \frac{1}{\theta} \left(\frac{v(w)}{v'(w)} - 1 - w \right). \tag{21}$$

Figure 1 investigates how bid-ask spreads affect firm value (scaled by capital) and the optimal investment rate. I use the same parameterization as BCW,[18] to which I add secondary market frictions and financing uncertainty as in Table 1. The left panel shows that scaled firm value decreases with the magnitude of the bid-ask spread, consistent with the results in Section 3.2. The right panel illustrates the optimal investment rate as a function of scaled cash reserves. As in BCW, the figure shows that the firm may have incentives to disinvest when its cash reserves are low, in which case financial constraints are more severe.[19] In the cash region characterized by positive investment, the optimal $i(w)$ is smaller if the bid-ask spread is wider. At the target cash level, the optimal investment rate is equal to 10.7% when the bid-ask spread is zero. If the bid-ask spread is equal to 60 basis points, the optimal investment rate is equal to 6.37%. Finally, if the bid-ask spread is equal to 120 basis points, the optimal investment rate is equal to 3.61%.

[18] Details are reported in Appendix A.5.

[19] When cash reserves are close to zero, the marginal value of cash increases, and firm value decreases—as a result, the first term in equation (21) decreases, and the investment rate can become negative.

The figure also confirms that the target cash level is smaller if the bid-ask spread is wider. Importantly, this analysis demonstrates that bid-ask spreads not only affect the firm's incentives to keep cash as a store of value, but also the incentives to accumulate capital, then giving rise to a substantial underinvestment problem.

3.3.3 Debt financing (bank credit lines)

Small firms typically do not have access to the corporate bond or commercial paper markets, and are more likely to tap debt financing by drawing funds from bank lines of credit. A credit line is a source of funding that the firm can access at any time up to a pre-established limit, which I denote by L. Whenever the credit limit is finite ($L < \infty$), the firm has a positive demand for cash. As shown by BCW, this is true for exogenous or endogenous (value-maximizing) L.[20] In this section, I assess the model results in the presence of this additional source of financing.

I follow BCW and assume that the firm pays a constant spread, β, over the risk-free rate on the amount of credit used. Because of this cost, it is optimal for the firm to tap the credit line only when cash reserves are exhausted. The firm then uses cash as the marginal source of financing if $c \in [0, C_V(L)]$ (the cash region), where $C_V(L)$ denotes the target cash level in this environment. Conversely, the firm draws funds from the credit line when $c \in [-L, 0]$ (the credit line region). Firm value satisfies (6) in the cash region, whereas it satisfies

$$(\rho + \delta\eta)\, V(c) = [(\rho + \beta)c + \mu]\, V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda\, [V(C_V) - V(c) - C_V + c] \qquad (22)$$

in the credit-line region. On top of the smooth-pasting and super-contact conditions at $C_V(L)$ similar to (8) and (9), the system of ODEs (6)–(22) is solved subject to the following boundary conditions. The first condition, $V(-L) = \max\, [\ell - L, 0]$, means that if $\ell \geq L$, the credit line is fully secured. Moreover, the conditions $\lim_{c \uparrow 0} V(0) = \lim_{c \downarrow 0} V(0)$

[20] Firms often face credit supply frictions that prevent them from taking the value-maximizing limit L. Endogenizing L is an interesting extension to understand the relation between stock liquidity and the firm's willingness to access bank credit, and I leave it for future research.

and $\lim_{c \uparrow 0} V'(0) = \lim_{c \downarrow 0} V'(0)$ guarantee continuity and smoothness at the point where the cash and the credit line regions are pasted.

Figure 2 studies the impact of bid-ask spreads when allowing the firm to access bank credit. I use the same parametrization in Table 1 and additionally set $L = 0.08$ and $\beta = 1.5\%$ (in line with Sufi, 2009). The figure shows that the effects of bid-ask spreads on corporate policies are similar irrespective of the firm's access to bank credit. Access to credit relaxes the precautionary need to keep cash and leads to a decrease in the target cash level. The presence of the bid-ask spread reduces this target level below the case with zero bid-ask spread. The probability of liquidation and payout as well as investment decisions are almost unchanged when allowing for firm's access to credit lines.

4 Endogenizing the bid-ask spread

Having analyzed how bid-ask spreads affect corporate financial policies, liquidation risk, investment, and firm value, I now derive the equilibrium bid-ask spread by assuming competition and participation frictions faced by liquidity providers. That is, in this section, η is derived endogenously—thus, not only it affects, but also reflects firm value.[21] In the following, I interchangeably use trading firms or liquidity providers.

4.1 Endogenous liquidity provision and corporate policies

Trading firms are liquidity providers that are active on both the bid and ask sides of transactions. As in Section 3, the ask price is equal to the fundamental value of the firm, $V(c)$, as trading firms sell stocks to non-liquidity-shocked investors on this side of transactions. On the bid side, trading firms buy stocks from liquidity-shocked investors. Bertrand competition among trading firms implies that the equilibrium bid-ask spread is

[21] A previous version of this paper focused on endogenizing the measure of active liquidity providers, yielding similar predictions. The results are available upon request.

determined by the zero-profit condition, which is given by:

$$-\delta\,(1-\eta)\;V(c;\eta,C_V(\eta))+\delta V(c;\eta,C_V(\eta))(1-\kappa)=\gamma \quad s.t. \quad \eta\le\chi \tag{23}$$

at any time. The first term on the left-hand side is the price at which trading firms buy the stock from liquidity-shocked shareholders. The second term is the price at which trading firms sell the stock to non-shocked investors net of their funding cost. The right-hand side of this equation is the participation cost borne by trading firms to maintain constant market presence. The equilibrium η cannot exceed χ, otherwise shocked shareholders would be better off holding the stock instead of selling it to trading firms. Notably, the endogenous bid-ask spread set by trading firms affects *and* depends on firm value. Using standard arguments, firm value satisfies the following HJB equation:

$$\begin{aligned}\rho V(c;\eta)=(rc+\mu)\,V'(c;\eta)+\frac{\sigma^2}{2}V''(c;\eta)+\lambda\sup_f[V(c+f;\eta)-V(c;\eta)-f]\\+\delta\left[(1-\min[\eta(c);\chi])V(c,\eta)-V(c,\eta)\right].\end{aligned} \tag{24}$$

The last term on the right-hand side is the loss borne by liquidity-shocked shareholders. The other terms in this equation admit an interpretation similar to equation (5). Similar to the case with constant η, it is optimal to raise funds up to the target level whenever financing opportunities arise, i.e., $f=C_V(\eta)-c$. Using this result and condition (23), equation (24) can be re-written as follows:

$$(\rho+\delta\kappa)V(c)=(rc+\mu)\,V'(c)+\frac{\sigma^2}{2}V''(c)+\lambda\,[V(C_V)-C_V+c-V(c)]-\gamma \tag{25}$$

whenever $\eta(c)<\chi$. Equation (25) implies that the frictions borne by liquidity providers are passed on to the liquidity-shocked shareholders. As a result, frictions affecting the provision of liquidity end up affecting firm value. Equation (25) is solved subject to conditions similar to those in Section 3, i.e. $V(0)=\ell$ and $\lim_{c\uparrow C_V}V'(c)=1$ at the liquidation threshold and at the target cash level, respectively. Again, the target cash level is identified by the super-contact condition, $\lim_{c\uparrow C_V} V''(c) = 0$.

Through equation (23), the equilibrium bid-ask spread that makes the zero-profit condition binding satisfies:

$$\eta(c) = \min\left[\chi, \frac{\gamma}{\delta V(c)} + \kappa\right]. \tag{26}$$

Equation (26) illustrates that larger participation or financing frictions faced by liquidity providers (γ and κ) lead to an increase in the equilibrium bid-ask spread both directly and through their effect on firm value (these effects are quantified in the next subsection). It also captures the real-world observation that the bid-ask spread is wider for smaller capitalization firms, as shown by Hasbrouck (2009) and Chung and Zhang (2014) among others.

Consider now the case in which the bid-ask spread binds at χ. Whenever $\eta = \chi$, investors are indifferent between selling the stock or keeping it. This is consistent with the real-world observation that bid-ask spreads do not increase unboundedly but, especially for small firms, trading volume wanes if bid-ask spreads grow too large. In these cases, firm value satisfies the following ODE:

$$\rho V(c) = (rc + \mu) V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda [V(C_V) - C_V + c - V(c)] - \delta\chi V(c). \tag{27}$$

Equation (26) suggests that η can become binding at χ if trading firms' participation fees or funding costs are sufficiently large, and firm value is sufficiently low (i.e., it can happen in a right interval of $c = 0$).[22] Whenever $\eta = \chi$ in a right interval of $c = 0$, continuity and smoothness determine how the two ODEs, equations (27) and (25), are pasted together (see Appendix B).

Implications When liquidity provision is endogenous, bid-ask spreads not only affect but are also affected by firm value. In fact, the drop in firm value analyzed in Section 3

[22] As shown in the appendix, there is at most one cash threshold $\underline{C} \in [0, C_V]$ such that: For $c \in [\underline{C}, C_V]$, the proportional bid-ask spread is $\eta(c) \in [0, \chi]$, which solves equation (23); for $c \in [0, \underline{C})$, a fraction δ of shareholders is indifferent between keeping the stock or bearing the bid-ask spread $\eta = \chi$.

(i.e., due to the costs borne by the firm shareholders when trading) is factored into the liquidity providers' zero-profit condition (equation (23)). Specifically, it leads liquidity providers to extract more rents from shocked investors (as a proportion of the value of their claim) to cover participation and funding costs. As a result, the bid-ask spread widens, so trading the stock becomes more expensive for shareholders. The model then shows that frictions borne by liquidity providers are passed on to shareholders and, through this channel, affect corporate outcomes.

Figure 3 and 4 show the endogenous bid-ask spread and firm value for different values of the participation fee γ and funding cost κ borne by liquidity providers. The top panels of these figures show that bid-ask spreads are wider when participation fees or funding costs faced by liquidity providers are greater. This result is consistent with the evidence in Anand et al. (2013), who show that liquidity suppliers extract more rents from investors when facing participation or funding costs, a pattern that is particularly strong for small-capitalization stocks. The result is also in line with Comerton-Forde et al. (2010), who show that variation in the liquidity of a given stock depends, at least in part, on market makers' financial constraints. The top and bottom panels of these figures also illustrate that bid-ask spreads are larger for less valuable firms.

Notably, the model shows that liquidity providers' participation frictions impact the equilibrium bid-ask spread through a direct and an indirect channel. Consider the effect of an increase in the frictions γ or κ. Such an increase implies that liquidity providers extract a relatively larger share of the value of the claim of shocked shareholders to remain active in the market of the stock. The bid-ask spread borne by shocked shareholders then widens (see equation (23)). This is the direct effect. Yet, this wider bid-ask spread then leads to a reduction in firm value (as shown in Section 3), which prompts liquidity providers to charge an even larger bid-ask spread. This is the indirect effect.

Quantitatively, an increase in γ from 0.7% to 0.8% would lead to a 9.1% increase in the bid-ask spread if firm value was independent of γ (i.e., if firm value was not affected by such an increase).[23] However, when accounting for its impact on firm value (i.e., the

[23]For the sake of fixing ideas, firm value is calculated at the midpoint of cash reserves, $c = C_V/2$, in

indirect effect described above), the bid-ask spread is 10.8% bigger as a result of the increase in γ. Similarly, an increase in κ from 0.3% to 0.4% would lead to an increase in the bid-ask spread by 12.2%. When accounting for the effect on firm value, the bid-ask spread increases by 14.4%. The joint impact of such direct and indirect effects provides an explanation as to why small firms' bid-ask spreads increase the most (i.e., much more than those of large firms) when liquidity providers' funding constraints tighten, see for example Anand et al. (2013) and Aragon and Strahan (2012).[24]

By affecting the equilibrium bid-ask spread, participation frictions faced by liquidity providers impact corporate policies, through the mechanism described in Section 3. Table 5 shows that larger γ or κ exacerbate the firm's financial constraints by increasing the firm's cost of internal and external financing. Thus, the target cash level decreases, cash is paid out more often to shareholders, and the probability of liquidation increases. Table 5 also shows that frictions borne by liquidity providers have a substantial impact on the firm's investment decisions, by leading to a sharp reduction in the maximum price that the firm is willing to pay to increase the cash flow drift from μ to μ_+.[25] Overall, frictions faced by liquidity providers have a substantial, detrimental effect on firm value (as quantified in the last column of Table 5).

Figure 5 further investigates the effects of γ or κ on the firm's probability of liquidation, external financing, and payout. It illustrates that the probability of liquidation increases with γ and κ and is greater than in the case with no bid-ask spreads. In addition, by leading to an increase in the bid-ask spread and, thus, in the firm's cost of external financing, frictions borne by liquidity providers lead to a sharp decrease in the firm's probability of external financing, which is lower than in the benchmark case in which the bid-ask spread is zero. Finally, by increasing the opportunity cost of cash, frictions borne by liquidity providers lead to an increase in the firm's payout probability.

these numerical examples.

[24]Conversely, because large firms are traded at small bid-ask spread and have a relatively easy access to several sources of financing, the reinforcing effect between financial constraints and trading frictions should be less important.

[25]The expression for the maximum zero-NPV cost is provided in Appendix B and is analogous to that reported in Proposition 4.

These results illustrate that frictions borne by liquidity providers are eventually passed on to investors and substantially affect corporate policies and value, consistent with Goldberg (2020). It is worth emphasizing that, whereas the economic mechanism described in the model can apply to any firm whose stock's bid-ask spread is non-negligible, it is highly relevant for small firms, that indeed face the largest bid-ask spreads in the cross-section and have uncertain access to outside financing. As a result, regulation affecting equity markets can affect the value and policies of these firms, as investigated below.

4.2 Applications

4.2.1 Firm-funded Designated Market Makers (DMM)

Theoretical models have shown that competitive market forces may lead to inefficient liquidity provision and suboptimal market outcomes (or market failures), see for instance Bessembinder, Hao, and Zheng (2015).[26] This issue has raised the attention of policy-makers. For example, the recommendations of the SEC Advisory Committee on Small and Emerging Companies are based on the idea that competitive market forces may break down when it comes to small or micro capitalization firms. Partially addressing this issue, several stock markets (like in Germany, France, Italy, the Netherlands, Sweden, and Norway) have contemplated a contract, whereby listed firms pay a DMM to maintain the bid-ask spread below a given (contractual) threshold and enhance the liquidity of the firm's stock. In this section, the model is extended to study the desirability of this policy provision from the perspective of small, financially constrained firms.

Consider the contract between a listed firm and a DMM. Similar to Bessembinder, Hao, and Zheng (2015), I assume that the DMM is required to keep the bid-ask spread within a specific width, in exchange for a rent that is paid by the firm. Consistently, I assume that the firm pays the DMM a periodic payment Γ to keep the bid-ask below a given level $\bar{\eta}$, which is assumed to be smaller than χ, to consider the relevant case. That

[26] Specifically, Bessembinder, Hao, and Zheng (2015) show that competitive liquidity provision in secondary markets is associated with reduced welfare and a discounted secondary market price that can potentially dissuade IPOs.

is, $\eta(c) \leq \bar{\eta} < \chi$ for any c. In this setup, the equilibrium bid-ask spread solves:

$$-\delta\,(1-\eta)\;V(c;\eta,C_V(\eta)) + \delta V(c;\eta,C_V(\eta))(1-\kappa) = \gamma - \Gamma \quad s.t. \quad \eta \leq \bar{\eta} < \chi. \tag{28}$$

The above condition differs from (23) in two main aspects. First, the participation fee borne by liquidity providers is partly financed by the firm (i.e, the right-hand side of this equation is just $\gamma - \Gamma$). Second, the contractual payment Γ guarantees that the equilibrium η cannot exceed $\bar{\eta}$ (see Appendix C). Intuitively, if $\bar{\eta}$ is smaller (meaning that DMM are required to keep the bid-ask spread more narrow), then the periodic fee Γ is larger (i.e., it is more costly to enter the DMM contract from the firm perspective).

By using the results in Section 4.1 together with condition (28), firm value is shown to satisfy the following ODE:

$$(\rho+\delta\kappa)V(c) = (rc+\mu-\Gamma)\,V'(c) + \frac{\sigma^2}{2}V''(c) + \lambda\,[V(C_V) - C_V + c - V(c)] - (\gamma-\Gamma). \tag{29}$$

This equation shows that the periodic flow paid by the firm to liquidity providers reduces the extent to which participation costs are passed on to shareholders (the last term on the right-hand side is $\gamma - \Gamma$, whereas it is just γ if the firm does not enter the DMM contract). However, the first term on the right-hand side of equation (29) illustrates that the payment to the DMM, Γ, drains the firm's periodic cash flow. The following result is shown in Appendix C.

Proposition 5 *Entering the DMM contract does not enhance the value of financially constrained firms, on net.*

Firms face the following tradeoff when deciding whether to enter the DMM contract. On the positive side, entering the DMM contract reduces the bid-ask spread borne by firm shareholders, which leads to a decrease in the cost of internal and external financing (as illustrated so far in the analysis). On the negative side, entering the DMM contract drains the firm's cash flows, which in turn makes the firm more financially constrained.

Intuitively, because the marginal value of cash is greater than one for financially constrained firms (see Lemma 6 in Appendix A), cash is more valuable inside the firm than if used to fund the DMM. As a result, the negative effect dominates the positive effect.

Figure 6 compares the equilibrium bid-ask spread and firm value when the firm does and does not enter the DMM contract.[27] The top panel shows that when the firm enters the DMM contract, the equilibrium bid-ask spread decreases, which is beneficial to the firm as it decreases its cost of capital. However, at the same time, the firm has to give up a fraction of its periodic flow of revenues to fund the DMM. On net, the bottom panel of Figure 6 shows that firm value is quite unchanged, being slightly smaller if the firm enters the DMM contract, consistent with Proposition 5.

4.2.2 Financial transaction taxes (FTT)

In the past decade, the European Commission has discussed the introduction of a proportional financial transaction tax (FTT) on round-trip transactions (i.e., "Tobin tax"). The main goal of the FTT is to prevent short-term speculation and limit volatility in financial markets. France introduced a 0.2% FTT in 2012, followed by Italy introducing a 0.1% tax in 2013.[28] In March 2019, the U.S. Democrats suggested to impose a 0.1% tax on equity transactions in the attempt to curb high-frequency trading.

Practically, the FTT is a surcharge on the cost of trading borne by the investors—liquidity providers are exempted from this tax, see Colliard and Hoffmann (2017). While abstracting from general equilibrium aspects, the economic mechanism presented in this paper suggests that this policy proposal would be detrimental to small firms. Intuitively, if investors face increased costs of trading equities because of the FTT, the issuing firms ultimately need to promise a larger return to shareholders. To see this more formally, denote the FTT by ω. Shareholders bear the cost $\eta(c)+\omega$ when selling the stock,[29] where

[27] When the firm does not enter the DMM contract, the bid-ask spread is solved as in Section 4.1.

[28] More precisely, transactions of shares issued by Italian resident companies are to be taxed at a 0.1% rate if executed on-exchange, and at a 0.2% rate if over-the-counter. See Coelho (2016) for additional details.

[29] For tractability, and consistently with our modeling of selling-side frictions only (as discussed in Section 2.1), I assume that the selling party is bearing the tax.

$\eta(c)$ is endogenously set by liquidity providers. As a result, the liquidity providers' zero-profit condition is similar to equation (23) but, differently, the constraint $\eta(c) < \chi - \omega$ needs to hold to make shareholders willing to sell the stock.[30] If $\eta(c) < \chi - \omega$ holds, firm value solves the following ODE (details are in Appendix C):

$$(\rho + \delta\kappa + \delta\omega)V(c) = (rc + \mu)\, V'(c) + \frac{\sigma^2}{2}V''(c) + \lambda\left[V(C_V) - C_V + c - V(c)\right] - \gamma. \quad (30)$$

That is, even if the FTT is formally imposed on investors only, it also affects the dynamics of firm value, as it effectively leads to an increase in the return required by the investors (see the left-hand side of equation (30)).

Figure 7 shows the effect of a 0.1% FTT on the equilibrium bid-ask spread and firm value. Because the FTT is ultimately borne by the investors, it increases the return that the firm needs to promise to its shareholders, which in turn reduces firm value. As illustrated in Section 3, the firm is then more subject to forced liquidation, keeps less cash, and forgoes profitable investment opportunities. Furthermore, through the mechanism illustrated in Section 4.1, this drop in firm value enters the liquidity providers' zero-profit condition. The reduction in firm value implies that competitive trading firms need to extract a larger fraction of firm value to cover the participation cost. In fact, the top panel shows that the endogenous bid-ask spread set by competitive liquidity providers is greater in the presence of the FTT.

Overall, this analysis suggests that the FTT would have detrimental real effect on small, financially constrained firms. That said, the analysis is silent on the welfare gains arising from this tax—it abstracts from speculative trading and other forces that could make the tax desirable. The model suggests that if a FTT is beneficial to warrant the stability of financial markets, then a provision making the magnitude of the FTT contingent on a firm's market capitalization could help relieve the potentially harmful real effects illustrated above.

[30] If the opposite inequality held, shocked shareholders would bear the cost of the liquidity shock.

5 Concluding remarks

Two key features characterize small firms: They face financing frictions, and trading their stocks entails non-negligible bid-ask spreads. This paper develops a model that studies the intertwined relation between these characteristics and analyzes their real effects. The model shows that bid-ask spreads increase the firms' cost of equity *and* the opportunity cost of cash. As such, they make firms more financially constrained—as they lead firms to keep smaller cash reserves and to raise external financing less frequently—and more exposed to forced liquidations. These firms also face a severe underinvestment problem, as the additional return required by the investors erodes the profitability of investment opportunities. Overall, these effects decrease firm value. These results are shown to be robust to alternative modeling assumptions regarding the firm's financing menu and financing frictions, as well as to a setup in which firms endogenously choose capital and cash accumulation.

The model also shows that when liquidity providers face participation frictions and set the bid-ask spread competitively, this drop in firm value feeds back into the magnitude of the bid-ask spread. This mechanism implies that frictions faced by liquidity providers are passed on to investors and, through this channel, have an important impact on the policies, values, and survival rates of small firms. The model then illustrates a channel through which frictions in the financial sector propagate to the corporate sector. More generally, the model suggests that the architecture of secondary market transactions has a prime effect on corporate decisions, especially for firms that are faced with large bid-ask spread and severe financing frictions.

Appendices

A Proof of the results in Section 3

Throughout the Appendix, I define the quantity $\Phi \equiv \delta\eta$ to ease the notation. I start by proving that $V(c)$ is increasing and concave for any $c < C_V$.

Lemma 6 *$V'(c) > 1$ and $V''(c) < 0$ for any $c \in [0, C_V)$.*

Proof. Simply differentiating the following equation (which is equivalent to equation (6))

$$(\rho + \lambda + \Phi)\ V(c) = V'(c)\,(rc + \mu) + \frac{\sigma^2}{2} V''(c) + \lambda\Big[V(C_V) - C_V + c\Big], \tag{31}$$

one gets

$$(\rho + \lambda + \Phi - r)\,V'(c) = V''(c)\,(rc + \mu) + \frac{\sigma^2}{2} V''(c) + \lambda.$$

By the conditions $V'(C_V) = 1$ and $V''(C_V) = 0$, it follows that $V'''(C_V) = \frac{2}{\sigma^2}\,(\rho + \Phi - r) > 0$ as $r < \rho$, meaning that there exists a left neighborhood of C_V such that for any $c \in (C_V - \epsilon, C_V)$, with $\epsilon > 0$, the inequalities $V'(c) > 1$ and $V''(c) < 0$ hold. Toward a contradiction, I assume that $V'(c) < 1$ for some $c \in [0, C_V - \epsilon]$. Then there exists a point $C_c \in [0, C_V - \epsilon]$ such that $V'(C_c) = 1$ and $V'(c) > 1$ over (C_c, C_V), so

$$V(C_V) - V(c) > C_V - c \tag{32}$$

for any $c \in (C_c, C_V)$. For any $c \in (C_c, C_V)$ it must be also that

$$V''(c) = \frac{2}{\sigma^2}\left\{(\rho + \lambda + \Phi)\,V(c) - [rc + \mu]V'(c) - \lambda(V(C_V) + c - C_V)\right\}$$

Using (32), jointly with $V(C_V) = \frac{rC_V + \mu}{\rho + \Phi}$, it follows that

$$V''(c) < \frac{2}{\sigma^2}\left\{(\rho + \Phi)(V(C_V) + c - C_V) - rc - \mu\right\} = \frac{2}{\sigma^2}(c - C_V)(\rho + \Phi - r) < 0.$$

This means that $V'(c)$ is decreasing for any $c \in (C_c, C_V)$, which contradicts $V'(C_c) = V'(C_V) = 1$. It follows that C_c cannot exists. So, $V'(c) > 1$ and $V''(c) < 0$ for any $c \in [0, C_V)$, and the claim follows. ■

A.1 Proof of Proposition 1

In this section, I express the function $V(c)$ as a function of X, denoting the threshold satisfying $V'(X, X) - 1 = V''(X, X) = 0$. To prove the claim, I exploit the following auxiliary results.

Lemma 7 *The function $V(c, X)$ is decreasing in the payout threshold X.*

Proof. To prove the claim, I take $X_1 < X_2$, and I define the auxiliary function $k(c) = V(c, X_1) - V(c, X_2)$, that satisfies

$$(\rho + \Phi + \lambda)k(c) = (rc + \mu)k'(c) + 0.5\sigma^2 k''(c) + \lambda(X_1 - X_2)[r/(\rho + \Phi) - 1] \tag{33}$$

for any $c \in [0, X_1]$. By previous result and straightforward calculations, the function is positive at X_2 as $k(X_2) = (X_1 - X_2)[r/(\rho+\Phi) - 1] > 0$. By the definition of X_1 and X_2, the function $k(c)$ is decreasing and convex for $c \in [X_1, X_2)$. Therefore, $k(X_1) > 0$. Note that the function cannot have a negative local minimum on $[0, X_1]$ because the last term on the right hand side of (33) is positive. In addition, the function $k'(c)$ does not have neither a positive local maximum nor a negative local minimum, otherwise the equation $(\rho+\Phi+\lambda-r)k'(c) = (rc+\mu)k''(c)+0.5\sigma^2 k'''(c)$ would not hold (respectively $k'(c) > 0 = k''(c) > k'''(c)$ and $k'(c) < 0 = k''(c) < k'''(c)$ at a positive maximum and at a negative minimum). As k is convex at X_1, this means that k' is increasing at X_1, and therefore it must be negative for any $c \in [0, X_1]$. Jointly with $k(X_1) > 0$, this means that $k(c) > 0$ for any $c \in [0, X_2]$. The claim follows. ■

Lemma 8 *For a given payout threshold X and two given $\eta_1 > \eta_2$, $V(c, X, \eta_2) > V(c, X; \eta_1)$ holds for any $c \in [0, X]$.*

Proof. I define $\Phi_i = \delta\eta_i$ with $i = 1, 2$, and the auxiliary function $h(c) = V(c, X; \eta_2) - V(c, X; \eta_1)$. I need to prove that, for a given payout threshold X, $h(c) > 0$ for any $c \in [0, X]$. At X, the function is positive as

$$h(X) = (rX + \mu)\left(\frac{1}{\rho + \Phi_2} - \frac{1}{\rho + \Phi_1}\right) = (rX + \mu)\frac{\Phi_1 - \Phi_2}{(\rho + \Phi_1)(\rho + \Phi_2)} > 0,$$

because $\Phi_1 > \Phi_2$ as $\eta_1 > \eta_2$, and $h'(X) = h''(X) = 0$. In addition, the function satisfies

$$[rc + \mu]h'(c) + \frac{\sigma^2}{2}h''(c) - (\rho + \lambda + \Phi_2)h(c) + \lambda h(X) = (\Phi_2 - \Phi_1)V(c, X; \chi_1)$$

and the right hand side is negative. Differentiating gives $[rc+\mu]h''(c) + \frac{\sigma^2}{2}h'''(c) - (\rho + \lambda + \Phi_2 - r)h'(c) = (\Phi_2 - \Phi_1)V_s'(c, X; \chi_1)$. At X, I get $\frac{\sigma^2}{2}h'''(X) = \Phi_2 - \Phi_1$, meaning that $h'''(X) < 0$. This means that the second derivative is decreasing in a neighbourhood of X, so one has $h''(c) > 0$ in a left neighbourhood of X. In turn, this means that $h'(c)$ is increasing in such a neighbourhood of X, then implying that $h'(c) < 0$ in a left neighbourhood of X. Now I need to prove that the function is decreasing for any c smaller that X. Note that, by the ODE above, $h'(c)$ cannot have a negative local minimum. As $h'(X) = 0$ and it is negative and increasing in a left neighbourhood of X, this means that $h'(c)$ should be negative for any $c < X$, so $h(c)$ is always decreasing. As it is positive at X, it means that it should be always positive, so $h(c) > h(X) > 0$ so it is positive for any $c < X$. ■

Exploiting the results above, I can prove the following lemma.

Lemma 9 *For any $\eta_1 > \eta_2$, $C_V(\eta_1) < C_V(\eta_2)$.*

Proof. The payout thresholds $C_V(\eta_1)$ and $C_V(\eta_2)$ are the unique solution to the boundary conditions $V(0, C_V(\eta_2); \eta_2) - \ell = 0 = V(0, C_V(\eta_1); \eta_1) - \ell$. Exploiting the result in Lemma 8, I now take, for instance, $X = C_V(\eta_1)$. It then follows that

$$V(0, C_V(\eta_1); \eta_2) - \ell \ > \ 0 \ = \ V(0, C_V(\eta_1); \eta_1) - \ell.$$

As V is decreasing in the payout threshold, this means that $C_V(\eta_1) < C_V(\eta_2)$ to get the equality $\ell - V(0, C_V(\eta_2); \eta_2) = 0$. The claim follows. ■

The next results stem from Lemma 9.

Corollary 10 *When the bid-ask spread is positive, the target cash level is lower than in the benchmark case with no bid-ask spread, i.e.* $C_V < C^*$.

Note also that all the results in this section can be extended for two parameters $\delta_1 > \delta_2$. The following result is then straightforward.

Corollary 11 *For any* $\delta_1 > \delta_2$, $C_V(\delta_1) < C_V(\delta_2)$.

A.2 Proof of Proposition 2

Using the insights from Dixit and Pyndick (1994), the dynamics of $P_p(c, X)$ are given by

$$P_p'(c)(rc + \mu) + \frac{\sigma^2}{2} P_p''(c) - \lambda \ P_p(c) = 0$$
$$\text{s.t. } P_p(0) = 0, \quad P_p(X) = \ 1.$$

The first boundary condition implies that when the controlled cash process is absorbed at zero, the firm liquidates and the payout probability is zero. The second boundary condition is obvious given that cash is paid out at X. The following lemma shows that greater bid-ask spreads are associated with larger payout probability.

Lemma 12 *For any* $\eta_1 > \eta_2$, $P_p(c, C_V(\eta_1)) \geq P_p(c, C_V(\eta_2))$.

Proof. By Lemma 9, $C_V(\eta_1) < C_V(\eta_2)$. To ease the notation throughout the proof, I define $X_1 \equiv C_V(\eta_1)$ and $X_2 \equiv C_V(\eta_2)$. Consider the function

$$h(c) = \ P_p(c, X_1) - P_p(c, X_2).$$

Because of the boundary conditions at zero and X_1, $h(0) = \ 0$ and $h(X_1) = 1 - P_p(c, X_2) > 0$. This means that the function is null at zero, and positive at C_V. Note that $h(c)$ cannot have neither a positive local maximum ($h(c) > 0$, $h'(c) = 0$, $h''(c) < 0$) nor a negative local minimum ($h(c) < 0$, $h'(c) = 0$, $h''(c) > 0$) on $[0, X_1]$, as otherwise the equation $h''(c)\frac{\sigma^2}{2} + h'(c)[rc + \mu] - \lambda h(c) = 0$ would not hold. Therefore, the function must be always positive and increasing over the relevant interval, and the claim follows. ■

The result below is a straightforward consequence of Lemma 12 and the fact that, in the absence of trading costs, $\eta = 0$ (or $\delta = 0$).

Corollary 13 *When trading the firm's stock is costly, the payout probability* P_p *is larger than in the benchmark case with no trading costs, i.e.* $P_p(c, C^*) < P_p(c, C_V)$.

These results can be extended for two parameters $\delta_1 > \delta_2$, as follows.

Corollary 14 *For any* $\delta_1 > \delta_2$, $P_p(c, C_V(\delta_1)) \geq P_p(c, C_V(\delta_2))$.

A.3 Proof of Proposition 3

I derive the results regarding the probability of liquidation $P_l(c, X)$, because the probability of external financing is just $P_f(c, X) = 1 - P_l(c, X)$. Using standard methods (see e.g., Dixit and Pyndick, 1994), the dynamics of $P_l(c, X)$ are given by

$$P_l'(c)(rc + \mu) + \frac{\sigma^2}{2} P_l''(c) - \lambda \, P_l(c) = 0$$

$$\text{s.t. } P_l(0) = 1 \tag{34}$$

$$P_l'(X) = 0, \tag{35}$$

where the first boundary condition is given by the definition of P_l, while the second boundary condition is due to reflection at the payout threshold.

Now I prove that the probability of liquidation is higher when the firm's stocks are illiquid (i.e, the bid-ask spread associated with the firm stock is positive). To do so, I first prove that the probabilities $P_l(c, C^*)$ and $P_l(c, C_V)$ are decreasing and convex in c. In the following, I employ the generic function $P_l(c, X) \equiv P_l(c)$.

Lemma 15 *The probability $P_l(c, X)$ is decreasing and convex for any $c \in [0, X]$.*

Proof. To prove the claim, I exploit arguments analogous to those of Lemma 6. As $P_l'(X) = 0$ and $P_l(X) \geq 0$, it must be that $P_l''(X) > 0$. Then, there exists a left neighbourhood of X, $[X - \epsilon, X]$ with $\epsilon > 0$, over which $P_l'(c) < 0$ and $P_l''(c) > 0$. Toward a contradiction, suppose that there exists some $c \in [0, X - \epsilon]$ where $P_l'(c) > 0$. Then, there should be a $\bar{C}$ such that $P_l'(\bar{C}) = 0$, while $P_l'(c) < 0$ for $c \in [\bar{C}, X]$. For any $c \in [\bar{C}, X]$ it must be that

$$P_l''(c) = \frac{2}{\sigma^2} \left[\lambda P_l(c) - P_l'(c)(rc + \mu) \right] > \frac{2}{\sigma^2} \lambda P_l(X) > 0 \, .$$

Then, $P_l''(c) > 0$ for any $c \in [\bar{C}, X]$ means that $P_l'(c)$ is always increasing on $c \in [\bar{C}, X]$, contradicting $P_l'(\bar{C}) = P_l'(X) = 0$. The claim follows. ■

Now I prove that $P_l(c, C_V) \geq P_l(c, C^*)$.

Lemma 16 *For any $\eta_1 > \eta_2$, $P_l(c, C_V(\eta_1)) \geq P_l(c, C_V(\eta_2))$.*

Proof. By Lemma 9, $C_V(\eta_1) < C_V(\eta_2)$. To ease the notation throughout the proof, I define $X_1 \equiv C_V(\eta_1)$ and $X_2 \equiv C_V(\eta_2)$. By Lemma 15, the functions $P_l(c, X_1)$ and $P_l(c, X_2)$ are positive, decreasing and convex over the interval of definition. I define the auxiliary function

$$h(c) = P_l(c, X_1) - P_l(c, X_2).$$

Note that $h(c)$ cannot have neither a positive local maximum ($h(c) > 0$, $h'(c) = 0$, $h''(c) < 0$) nor a negative local minimum ($h(c) < 0$, $h'(c) = 0$, $h''(c) > 0$) on $[0, X_1]$, as otherwise the equation $h''(c)\frac{\sigma^2}{2} + h'(c)[rc + \mu] - \lambda h(c) = 0$ would not hold. In addition, $h(0) = 0$, and $h'(X_1) = -P_l'(c, X_2) > 0$ because of the boundary conditions at zero and at X_1. This means that the function is null at the origin, and increasing at C_V. Toward a contradiction, assume that $h(X_1)$ is negative. This would imply the existence of a negative local minimum, given that the function is null at zero and it is increasing at X_1. This cannot be the case as argued above, contradicting that $h(X_1) < 0$. Therefore, the function must be always positive, and the claim follows. ■

The result below is a straightforward consequence of Lemma 16 and the fact that, in the absence of trading costs, $\eta = 0$ (or $\delta = 0$).

Corollary 17 *When the bid-ask spread associated with the firm's stock is positive, the probability of liquidation P_l is larger than in the case in which the bid-ask spread is zero, i.e. $P_l(c, C^*) < P_l(c, C_V)$.*

These results can be extended to the case in which the parameter δ is varied.

Corollary 18 *For any $\delta_1 > \delta_2$, $P_l(c, C_V(\delta_1)) \geq P_l(c, C_V(\delta_2))$.*

A.4 Proof of Proposition 4

I exploit the dynamic programming result in Décamps and Villeneuve (2007) and HMM, establishing that the growth option has a non-positive NPV if and only if $V(c) > V_+(c - I)$ for any $c \geq 0$, where I denote by $V_+(c - I)$ the value of the firm after investment. To prove the claim, I rely on the following lemma.

Lemma 19 *$V(c) \geq V_+(c - I)$ for any $c \geq I$ if and only if $I \geq I_V$, where I_V satisfies the expression reported in Proposition 4.*

Proof. I define $\bar{c} = \max[C_V, I + C_{V+}]$. The inequality $V(c) \geq V_+(c - I)$ for $c > \bar{c}$ means that $c - C_V + V(C_V) \geq c - C_{V+} - I + V_+(C_{V+})$. Using the definition of I_V, the former inequality is equivalent to the inequality $I \geq I_V$, by straightforward calculations.

To prove the sufficient condition, I can just prove that $V(c) \geq V_+(c - I_V)$ for any $c \geq I_V$. I exploit the inequalities $C_V < C_{V+} + I_V$ and $\mu_+ - \mu - rI_V > 0$ (these inequalities stem from a slight modification of Lemma C.3 in HMM, so I omit the details). For $c \geq C_V$, the following inequality

$$V_+(c - I_V) \leq V_+(C_{V+}) + c - I_V - C_{V+} = c - C_V + V(C_V) = V(c)$$

holds. The first inequality is due to the concavity of V_+. The first equality is given by the definition of I_V, whereas the second equality is due to the linearity of V above C_V. I now need to prove the result for $c \in [I_V, C_V]$. To this end, I define the auxiliary function $u(c) = V(c) - V_+(c - I_V)$. The function $u(c)$ is positive at C_V as argued above, $u'(C_V) < 0$ and $u''(C_V) > 0$. On the interval of interest it satisfies

$$\begin{aligned}(\rho + \Phi + \lambda)u(c) =& (rc + \mu)u'(c) + \frac{\sigma^2}{2}u''(c) + (\mu + rI_V - \mu_+)V_+'(c - I_V) \\ &+ \lambda(V(C_V) - C_V - V_+(C_{V+}) + C_{V+} + I_V)\end{aligned}$$

where the last term on the right hand side is zero by the definition of I_V, while the third term is negative. Then, the function cannot have a positive local maximum here, because otherwise $u(c) > 0$, $u''(c) < 0 = u'(c)$, and the ODE above would not hold. Jointly with the fact that $u(C_V)$ is positive, decreasing and convex means that the function is always decreasing on this interval. Then, $u(c)$ is also always positive, and the claim holds. ■

A.5 Proof of the results in Section 3.3.2

This section reports additional calculations for the alternative setup reported in Section 3.3.2, investigating an environment with endogenous capital accumulation and continuous investment. Using standard arguments, firm value satisfies the following equation:

$$\rho V(C,K) = \max_i (i-d)KV_K + \left[\left(\mu - i - \frac{\theta i^2}{2}\right)K + rC\right]V_C + \frac{1}{2}K^2\sigma^2 V_{CC} \tag{36}$$

$$+ \delta\left[\Phi(C,K) - V(C,K)\right] + \lambda\left[V(C^*,K) - C^* - V(C,K) + C\right] \tag{37}$$

where $\delta\Phi(C,K)$ denotes the aggregate claim of shocked shareholders in this extended setup. Conjecture that $V(C,K) = v(w)K$, where $w = C/K$ denotes scaled cash reserves. Also, let us denote by W^* the target scaled cash reserves. Substituting in the above equation gives

$$(\rho + \delta\eta)v(w) = \max_i (i-d)(v - wv') + \left[\mu - i - \frac{\theta i^2}{2} + rw\right]v' + \frac{\sigma^2}{2}v''$$
$$+ \lambda\left[v(W^*) - W^* - v(w) + w\right] \tag{38}$$

Differentiating with respect to i gives the expression for the optimal investment rate

$$i(w) = \frac{v(w) - v'(w)(1+w)}{\theta v'(w)},$$

as reported in the main text. Substituting the expression for $i(w)$ into equation (38) and imposing boundary conditions similar to those in Section 3.1 gives scaled firm value. As in BCW, denote by l the liquidation value of assets scaled by the capital stock. Thus, firm value satisfies the condition $v(0) = l$ when scaled cash reserves reach zero. Moreover, at the threshold W^*, the following boundary conditions hold: $v'(W^*) = 1$ and $v''(W^*) = 0$.

Figure 1 analyzes scaled firm value and the optimal investment rates for different values of η, as reported in the legend. On top of the parameters specific to firm's financing frictions (λ) and to secondary market transactions (δ, χ)—for which I use the values reported in Table 1—I use the parameters pertaining to the neoclassic q framework reported in Table 1 of BCW. Specifically, the adjustment-cost parameter is set to 1.5, the rate of depreciation is equal to 10.07%, the liquidation value of capital is set to 0.9, the mean and the volatility of the productivity shock are respectively 0.18 and 0.09, the risk-free is equal to 0.06, and the opportunity cost of cash is equal to 0.01.

B Proof of the results in Section 4.1

Two separate cases are considered, depending on the relative magnitude of participation or funding costs borne by liquidity providers.

Case $\eta(c) < \chi$ for any c. This is the case if participation and funding costs are sufficiently low, so that the following inequality

$$\frac{\gamma}{\delta(\chi - \kappa)} \leq \ell \tag{39}$$

holds. When this is the case, firm value satisfies equation (25) for any $c < C_V$, and firm value is solved subject to the boundary condition at the liquidation threshold and at C_V as reported in the main text. As a straightforward extension of Lemma 6, it is possible to show that $V(c)$ is increasing and concave in cash reserves.

Case $\eta(c) = \chi$ for some c. This is the case if participation and funding costs are sufficiently large, so that condition (39) does not hold. Because firm value is increasing in c, there is at most one cash threshold $\underline{C} \in [0, C_V]$ such that the proportional bid-ask spread is $\eta(c) \in [0, \chi]$ for $c \in [\underline{C}, C_V]$ and solves equation (23). For $c \in [0, \underline{C})$, a fraction δ of shareholders is indifferent between keeping the stock or selling it at price $\chi V(c)$. Continuity and smoothness at $\underline{C}$ mean that the system of equations (25) and (27) is solved subject to the following conditions:

$$\lim_{c \uparrow \underline{C}} V(c) = \lim_{c \downarrow \underline{C}} V(c) \quad \text{and} \quad \lim_{c \uparrow \underline{C}} V'(c) = \lim_{c \downarrow \underline{C}} V'(c)$$

on top of the boundary conditions at the liquidation and payout threshold ($V(0) - \ell = \lim_{c \uparrow C_V} V'(c) - 1 = \lim_{c \uparrow C_V} V''(c) = 0$). In this case too, I show next that the value function is strictly monotone and concave over $0 \leq c < C_V$. Differentiating equation (25) gives the following ODE

$$(\rho + \lambda + \delta\kappa - r)\, V'(c) = V''(c)\,(rc + \mu) + \frac{\sigma^2}{2} V'''(c) + \lambda.$$

Jointly with the boundaries $V'(C_V) = 1$ and $V''(C_V) = 0$, this ODE implies that $V'''(C_V) > 0$, meaning that there exists a left neighborhood of C_V such that for any $c \in (C_V - \epsilon, C_V)$, with $\epsilon > 0$, the inequalities $V'(c) > 1$ and $V''(c) < 0$ hold. Toward a contradiction, I assume that $V'(c) < 1$ for some $c \in [0, C_V - \epsilon]$. Then, there should be a point $C_c \in [0, C_V - \epsilon]$ such that $V'(C_c) = 1$ and $V'(c) > 1$ over (C_c, C_V), so $V(C_V) - V(c) > C_V - c$ for any $c \in (C_c, C_V)$. The point C_c could belong either to the interval $[0, \underline{C}]$ or in the interval $[\underline{C}, C_V]$. I now discriminate between these two cases. If $\underline{C} < C_c < C_V$, it must be that for any $c \in (C_c, C_V)$

$$V''(c) = \frac{2}{\sigma^2} \left\{ (\rho + \lambda + \delta\kappa)\, V(c) - (rc + \mu) V'(c) + \gamma - \lambda[V(C_V) + c - C_V] \right\}.$$

Using $V(C_V) - V(c) > C_V - c$, jointly with $V(C_V) = \frac{rC_V + \mu - \gamma}{\rho + \delta\kappa}$, it follows that

$$V''(c) < \frac{2}{\sigma^2} \left\{ (\rho + \delta\kappa)(V(C_V) + c - C_V) - rc - \mu + \gamma \right\} = \frac{2}{\sigma^2}(c - C_V)(\rho + \delta\kappa - r) < 0.$$

This means that $V'(c)$ is decreasing for any $c \in (C_c, C_V)$, which contradicts $V'(C_c) = V'(C_V) = 1$. So, $V'(c) > 1$ for any $c \in [\underline{C}, C_V)$, so such C_c does not exist on $[\underline{C}, C_V]$.

I now consider the case $0 < C_c < \underline{C}$. Should such point C_c exist, the strict concavity of $V(c)$ over $[\underline{C}, C_V]$ means that there should be a maximum $C_m \in [C_c, \underline{C}]$ for the first derivative over the interval $(C_c, \underline{C})$, such that $V'(C_m) > 1$, $V''(C_m) = 0$ and $V'''(C_m) < 0$. Differentiating equation (27) gives

$$V''(c)[rc + \mu] + V'''(c)\frac{\sigma^2}{2} - V'(c)(\rho + \delta\chi - r) + \lambda(1 - V'(c)) = 0.$$

Then, $V'''(C_m)\frac{\sigma^2}{2} = (\rho + \delta\chi - r)V'(C_m) + \lambda(V'(C_m) - 1) > 0$, which contradicts the existence of such a maximum C_m for $V'(c)$. It follows that C_c cannot exists, and the claim follows.

Investment decision with endogenous liquidity provision. Following arguments similar to those in Appendix A.4, the firm finds it optimal to invest in the growth option if it has positive NPV. This is the case if $V(c) > V_+(c-I)$, where again I denote by $V_+(c-I)$ the value of the firm after investment. Thus, a straightforward modification of Lemma 19 implies that the zero-NPV investment (i.e., that makes the NPV of the project equal to zero) is given by the following expression:

$$I_V = \frac{\mu_+ - \mu}{\rho + \delta\kappa} - \left(1 - \frac{1}{\rho + \delta\kappa}\right)(C_{V+} - C_V) \tag{40}$$

where I denote by C_{V+} the target cash level after investment (i.e., when the cash flow drift is μ_+).

C Proof of the result in Section 4.2

C.1 Proof of the analysis in Section 4.2.1 (DMM)

Consider the zero-profit condition if the firm enters the DMM contract. The equilibrium payment Γ needs to guarantee that the maximum contractual bid-ask spread is $\bar{\eta} < \chi$. As shown in the analysis in Section 4.1, the bid-ask spread decreases as firm value increases. Thus, the minimum Γ that the firm needs to pay to the DMM solves the following equation:

$$\delta V(0)(1-\kappa) - \delta(1-\bar{\eta})V(0) + \Gamma = \gamma, \tag{41}$$

which gives $\Gamma^* = \gamma - \delta(\bar{\eta} - \kappa)V(0)$. In this environment, the bid-ask spread is given by the following expression:

$$\eta(c) = \frac{\gamma - \Gamma^*}{\delta V(c)} + \kappa \tag{42}$$

which indeed is lower than the equilibrium bid-ask when the firm does not enter the DMM contract. Next, I prove Proposition 5, showing that, on net, entering the DMM contract cannot increase firm value.

Proof. Comparing equation (29) with equation (25) illustrates that the two equations only differ for the additional term $-\Gamma^* V'(c) + \Gamma^*$ on the right-hand side. This expression can be rewritten as $\Gamma^*(1 - V'(c))$. As $V'(c) \geq 1$ for any $c < C_V$, the expression $\Gamma^*(1 - V'(c))$ is strictly negative for any $c < C_V$, which implies that it is suboptimal for a financially constrained firm to subsidize the DMM. ■

C.2 Proof of the analysis in Section 4.2.2 (FTT)

Because the FTT is ultimately borne by shareholders, it does not affect the liquidity providers' zero-profit condition directly. However, it does so indirectly through its impact on firm value.

That is, $\eta(c)$ solves a condition similar to (23) but subject to $\eta(c) + \omega < \chi$. Firm value satisfies:

$$\rho V(c;\eta) = (rc+\mu)\,V'(c;\eta) + \frac{\sigma^2}{2}V''(c;\eta) + \lambda \sup_f [V(c+f;\eta) - V(c;\eta) - f] \\ + \delta\left[(1-\min[\eta(c)+\omega;\chi])V(c,\eta) - V(c,\eta)\right]. \tag{43}$$

In this equation, the optimal refinancing size f replenishes the cash reserves, i.e., $f = C_V - c$ (as in the case with no FTT, see Section 4.1). The term $\min[\eta(c)+\omega;\chi]$ reveals that, all else equal, the presence of the FTT makes liquidity-shocked investors less willing to sell the stock (equivalently, more willing to keep the stock in the face of liquidity shocks). Whenever $\eta(c) < \chi - \omega$, firm value solves the following ODE:

$$(\rho + \delta\kappa + \delta\omega)V(c) = (rc+\mu)\,V'(c) + \frac{\sigma^2}{2}V''(c) + \lambda\,[V(C_V) - C_V + c - V(c)] - \gamma. \tag{44}$$

The left-hand side of this expression illustrates that the presence of the FTT further increases the return required by investors by the additional term $\delta\omega$. This additional return reduces firm value and implies that, all else being equal, trading firms extract larger concessions from investors. Whenever $\eta(c) > \chi - \omega$, firm value satisfies equation (27). The ODEs are solved to the same boundary conditions reported in Appendix B.

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Table 1: Benchmark parameters.

Symbol	Description	Value
	Firm	
ρ	Risk-free rate	0.02
r	Return on cash	0.01
μ	Cash flow drift	0.05
μ_+	Post-investment cash flow drift	0.06
σ	Cash flow volatility	0.12
ϕ	Recovery rate in liquidation	0.55
λ	Arrival rate of financing opportunities	0.75
L	Credit line limit	0.08
β	Credit line spread	0.015
	Stock transactions	
δ	Arrival rate of liquidity shocks	0.70
χ	Loss due to liquidity shocks	0.02
κ	Liquidity providers' funding cost	0.003
γ	Liquidity providers' participation cost	0.007
$\bar{\eta}$	DMM's bid-ask cap	0.008
ω	Financial transaction tax	0.001

Table 2: Effect of bid-ask spreads on corporate outcomes.

The table reports the target cash level, the average probability of external financing, of liquidation, and payout, the zero-NPV investment cost, and firm value (at the target cash level C_V) when varying the bid-ask spread (as reported in the first column).

Bid-ask spread (Basis points)	Target Cash Level	Financing Probability	Liquidation Probability	Payout Probability	Zero-NPV investment cost	Firm value
0	0.546	87.2%	12.8%	24.9%	0.504	2.773
10	0.534	86.9%	13.1%	25.4%	0.487	2.673
20	0.522	86.6%	13.4%	25.9%	0.471	2.580
30	0.510	86.3%	13.7%	26.4%	0.455	2.493
40	0.498	86.0%	14.0%	26.9%	0.441	2.411
50	0.486	85.6%	14.4%	27.5%	0.427	2.334
60	0.473	85.2%	14.8%	28.0%	0.414	2.262
70	0.460	84.8%	15.2%	28.7%	0.401	2.193
80	0.447	84.3%	15.7%	29.3%	0.389	2.128
90	0.432	83.8%	16.2%	30.1%	0.378	2.066
100	0.417	83.2%	16.8%	30.9%	0.366	2.006
120	0.383	81.6%	18.4%	32.9%	0.345	1.895
140	0.340	79.2%	20.8%	35.5%	0.323	1.792
160	0.285	74.7%	25.3%	39.4%	0.303	1.694
180	0.214	64.9%	35.1%	44.6%	0.285	1.599

Table 3: Bid-ask spreads and probability of forced liquidation.

The table reports the firm's probability of liquidation at different levels of cash reserves (i.e., at $C_V/2$, $C_V/4$, and $C_V/8$) when varying the bid-ask spread (as reported in the first column).

Bid-ask spread (Basis points)	$C_V/2$	$C_V/4$	$C_V/8$
0	1.97%	14.1%	37.6%
10	2.15%	14.7%	38.5%
20	2.34%	15.4%	39.3%
30	2.56%	16.1%	40.1%
40	2.79%	16.8%	41.0%
50	3.06%	17.5%	41.9%
60	3.35%	18.3%	42.9%
70	3.69%	19.2%	43.9%
80	4.08%	20.2%	45.0%
90	4.53%	21.2%	46.1%
100	5.08%	22.5%	47.4%
120	6.58%	25.4%	50.5%
140	9.09%	29.7%	54.5%
160	14.0%	36.6%	60.3%
180	25.3%	48.4%	69.2%

Table 4: Effect of bid-ask spreads when financing frictions are modeled as issuance costs.

The table reports the target cash level, the size of equity issuances, the zero-NPV investment cost, and firm value (calculated at the target cash level C_V) when varying the bid-ask spread (as reported in the first column). Proportional and fixed financing costs are respectively equal to $\epsilon = 0.06$ and $\omega = 0.01$ in the top panel, and equal to $\epsilon = 0.1$ and $\omega = 0.03$ in the bottom panel.

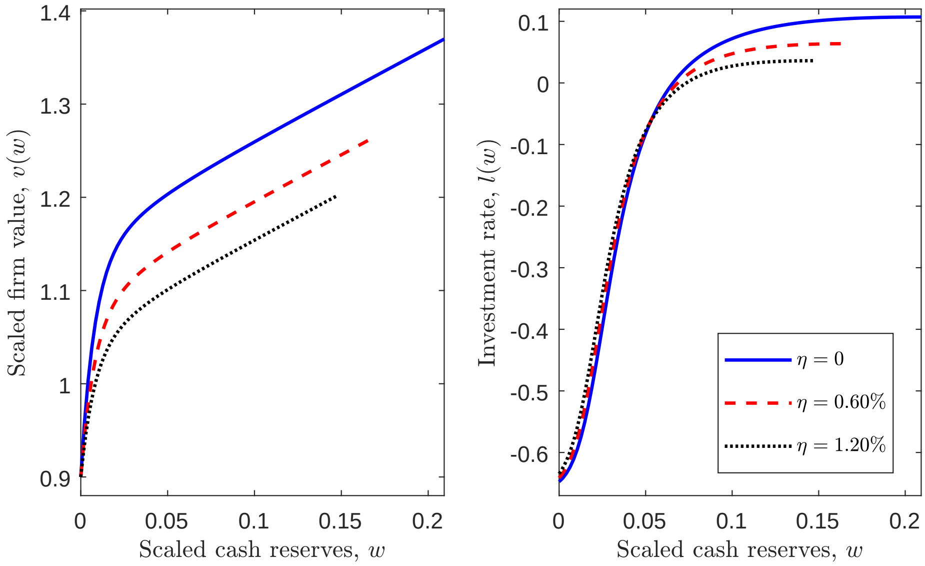
Bid-ask spread (Basis points)	Target Cash Level	Issuance Size	Zero-NPV investment cost	Firm value
$\epsilon = 0.06$, $\omega = 0.01$				
0	0.350	0.133	0.509	2.675
20	0.338	0.131	0.476	2.494
40	0.327	0.129	0.447	2.336
60	0.317	0.127	0.422	2.197
80	0.309	0.125	0.399	2.074
100	0.301	0.124	0.379	1.963
120	0.294	0.122	0.360	1.864
140	0.288	0.121	0.344	1.774
160	0.282	0.120	0.328	1.693
180	0.276	0.118	0.314	1.619
$\epsilon = 0.1$, $\omega = 0.03$				
0	0.438	0.178	0.513	2.719
20	0.424	0.175	0.480	2.534
40	0.411	0.173	0.452	2.373
60	0.401	0.171	0.426	2.232
80	0.391	0.169	0.403	2.106
100	0.382	0.167	0.383	1.993
120	0.374	0.165	0.365	1.892
140	0.367	0.164	0.348	1.801
160	0.360	0.162	0.333	1.718
180	0.354	0.161	0.319	1.642

Table 5: Liquidity providers' participation frictions and corporate outcomes.

The table reports the change in the target cash level, in the probability of liquidation, in the probability of payout, in the zero-NPV investment cost, and in firm value (calculated at the target cash level) when accounting for participation fees (γ) and funding costs (κ) borne by liquidity providers with respect to a benchmark environment with no trading frictions (in which the bid-ask spread is zero).

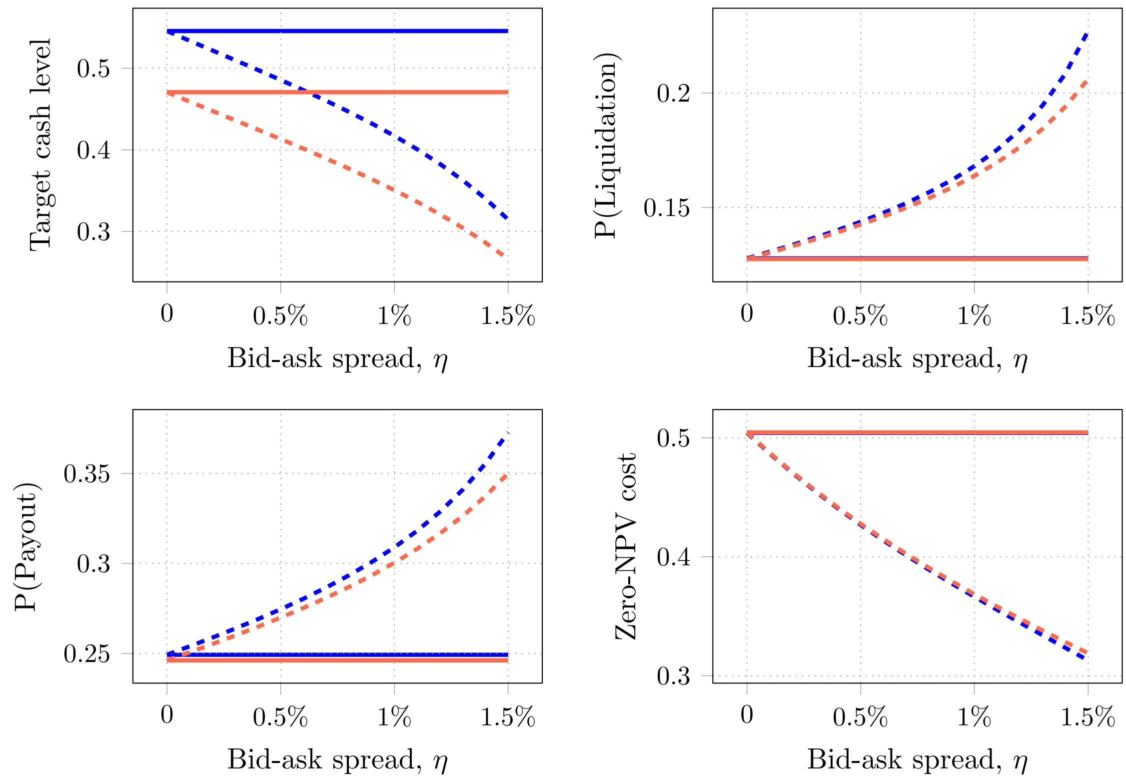
Participation friction	Target cash level (Δ%)	Liquidation probability (Δ)	Payout probability (Δ)	Zero-NPV investment cost (Δ%)	Firm value (Δ%)
$\gamma = 0.1\%$	-7.44%	1.04%	1.67%	-9.86%	-11.80%
$\gamma = 0.3\%$	-9.43%	1.35%	2.14%	-10.19%	-15.24%
$\gamma = 0.5\%$	-11.77%	1.73%	2.73%	-10.63%	-18.71%
$\gamma = 0.7\%$	-14.60%	2.22%	3.46%	-11.20%	-22.23%
$\gamma = 0.9\%$	-18.17%	2.89%	4.42%	-12.03%	-25.81%
$\kappa = 0.1\%$	-8.84%	1.25%	2.00%	-4.54%	-16.42%
$\kappa = 0.2\%$	-11.66%	1.71%	2.70%	-7.95%	-19.41%
$\kappa = 0.3\%$	-14.60%	2.22%	3.46%	-11.20%	-22.23%
$\kappa = 0.4\%$	-17.73%	2.80%	4.30%	-14.35%	-24.88%
$\kappa = 0.5\%$	-21.09%	3.49%	5.25%	-17.41%	-27.40%

Figure 1: Continuous investment.



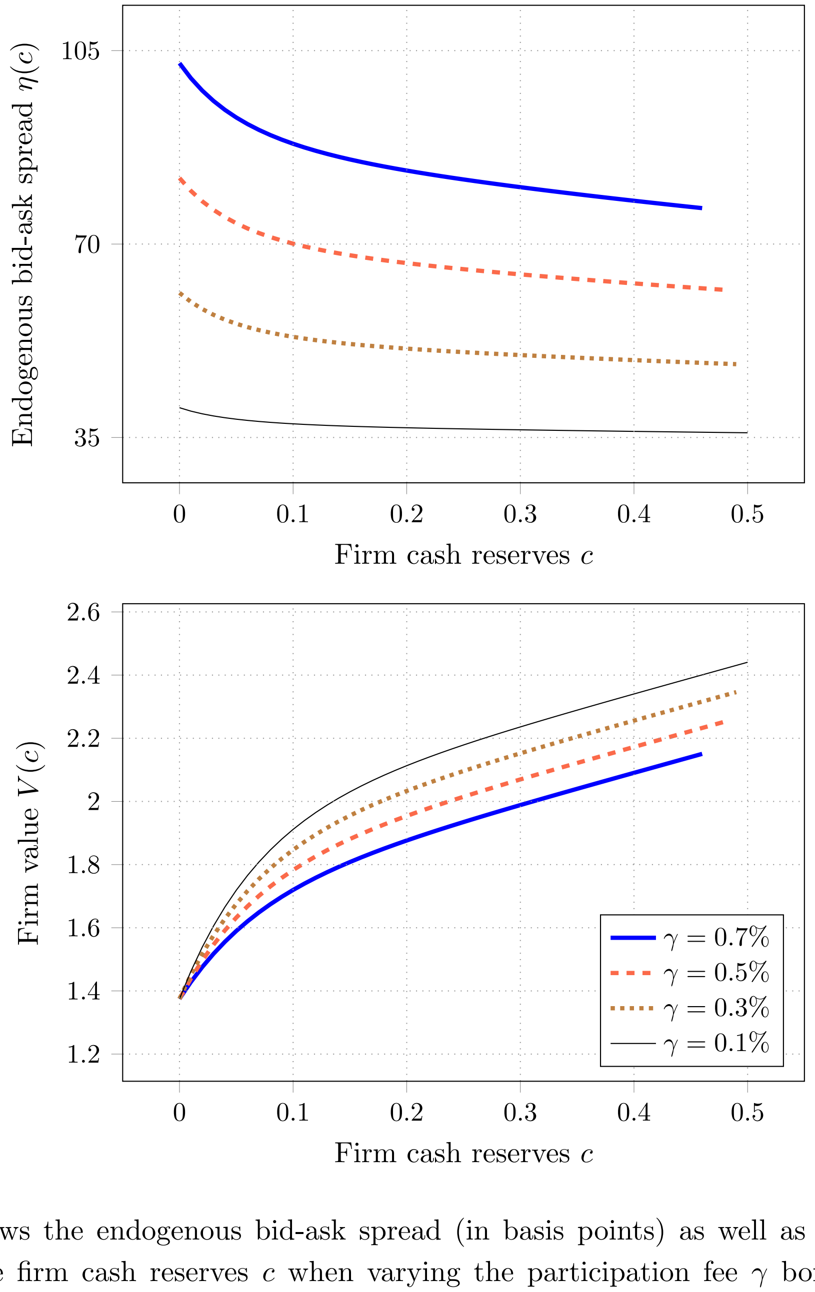
The left panel shows firm value scaled by capital as a function of scaled cash reserves, whereas the right panel shows the firm's optimal investment rate as a function of scaled cash reserves. The different lines correspond to different assumptions regarding the magnitude of the stock's bid-ask spread.

Figure 2: Allowing for credit line availability.



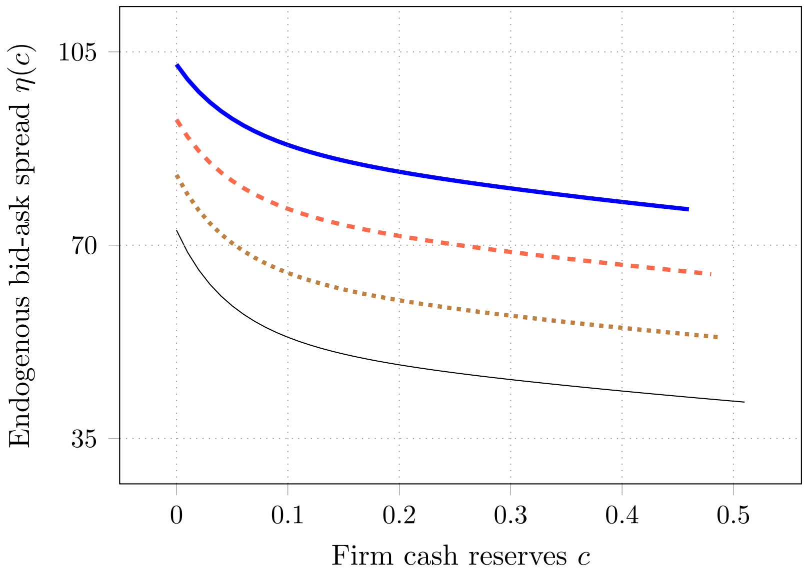
The figure shows the target level of cash reserves, the probability of liquidation, the probability of payout, and the maximum investment cost as a function of the bid-ask spread (η). The blue lines refer to a firm with no access to bank credit and when the bid-ask spread is zero (solid line) or positive (dashed line). The red lines refer to a firm having access to bank credit and when the bid-ask spread is zero (solid line) or positive (dashed line).

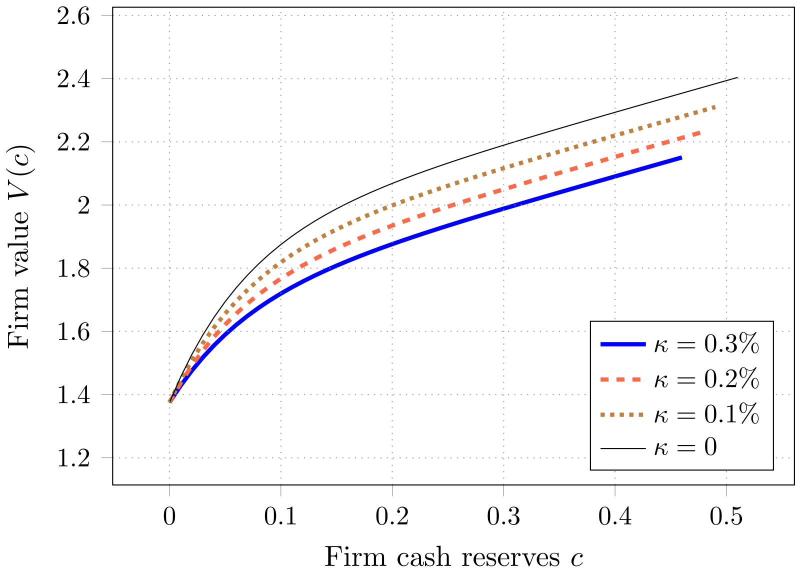
Figure 3: Endogenous liquidity provision and firm value when liquidity providers face participation frictions (I).



The figure shows the endogenous bid-ask spread (in basis points) as well as firm value as a function of the firm cash reserves c when varying the participation fee γ borne by liquidity providers.

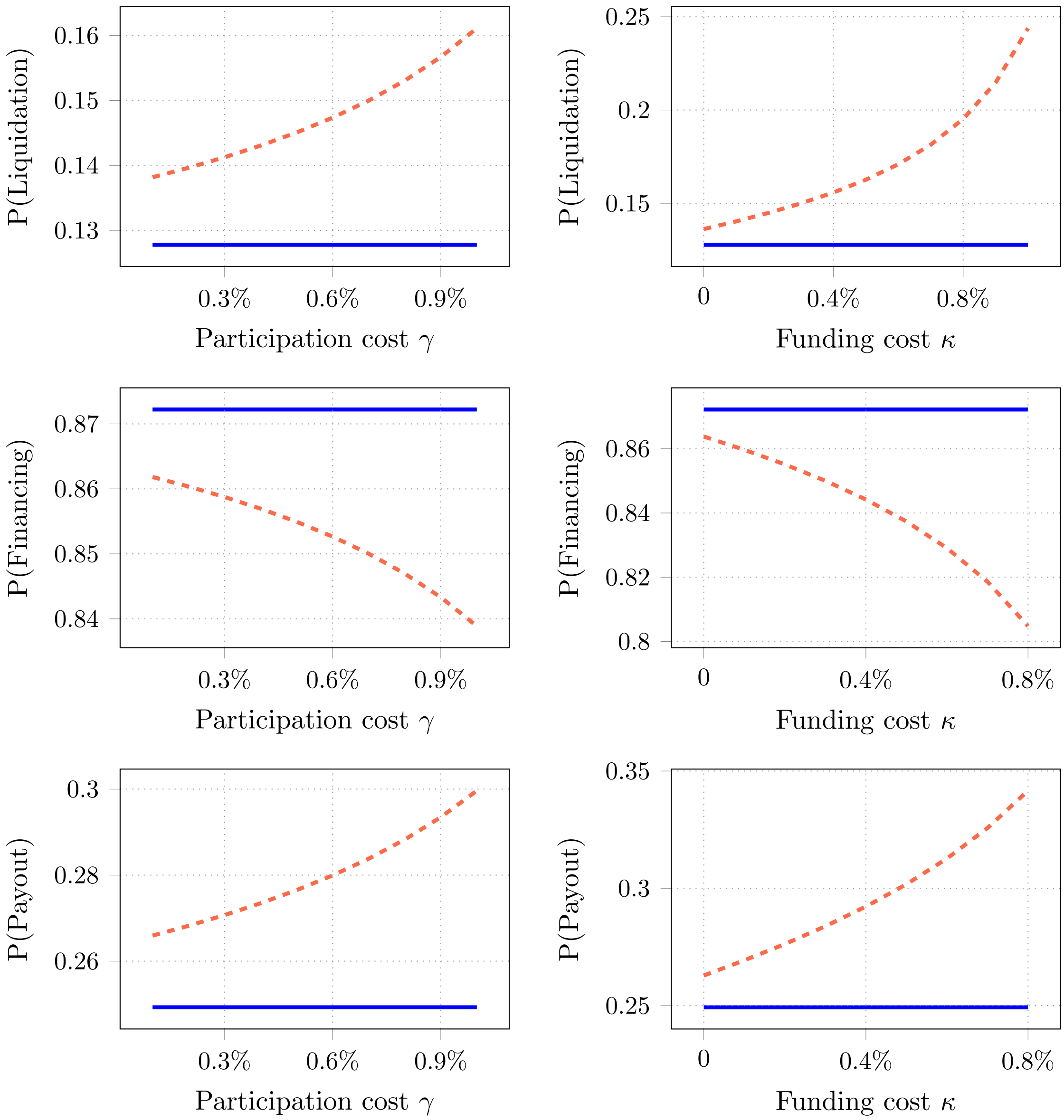
Figure 4: Endogenous liquidity provision and firm value when liquidity providers face participation frictions (II).





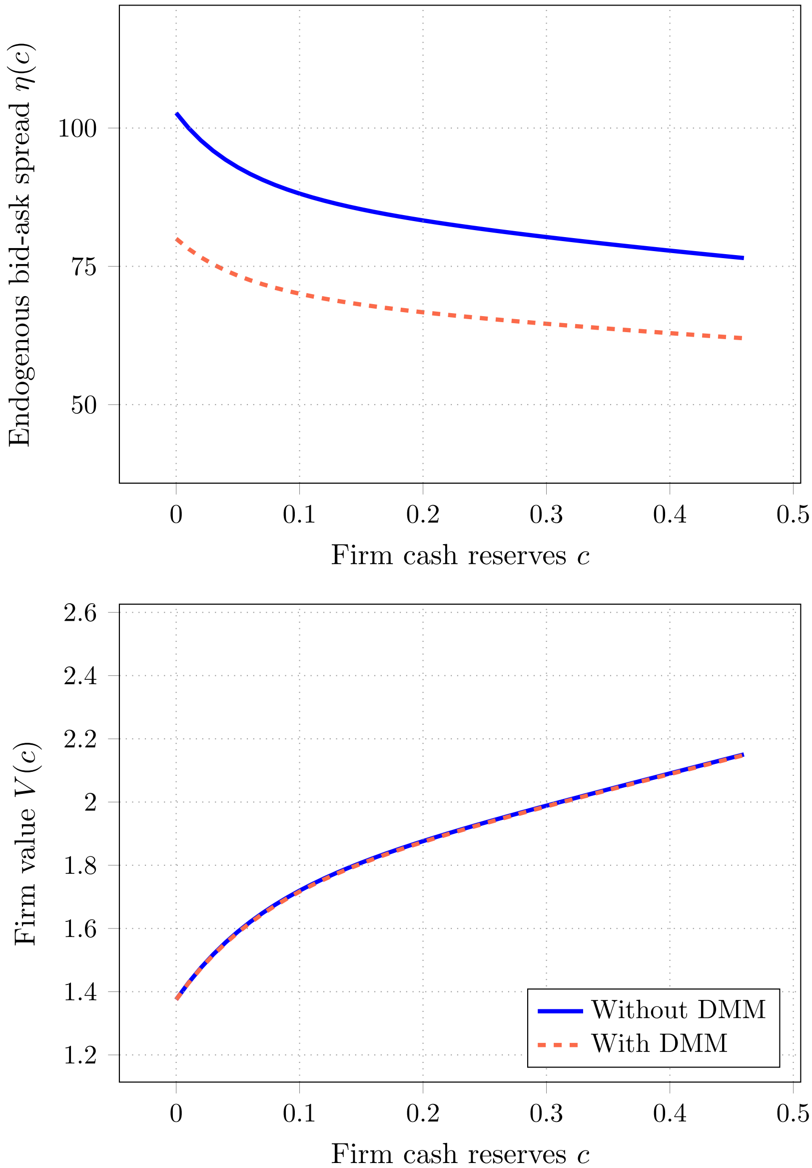
The figure shows the endogenous bid-ask spread (in basis points) as well as firm value as a function of the firm cash reserves c when varying the funding cost κ faced by liquidity providers.

Figure 5: The impact of participation frictions on the firm probability of liquidation, financing, and payout.



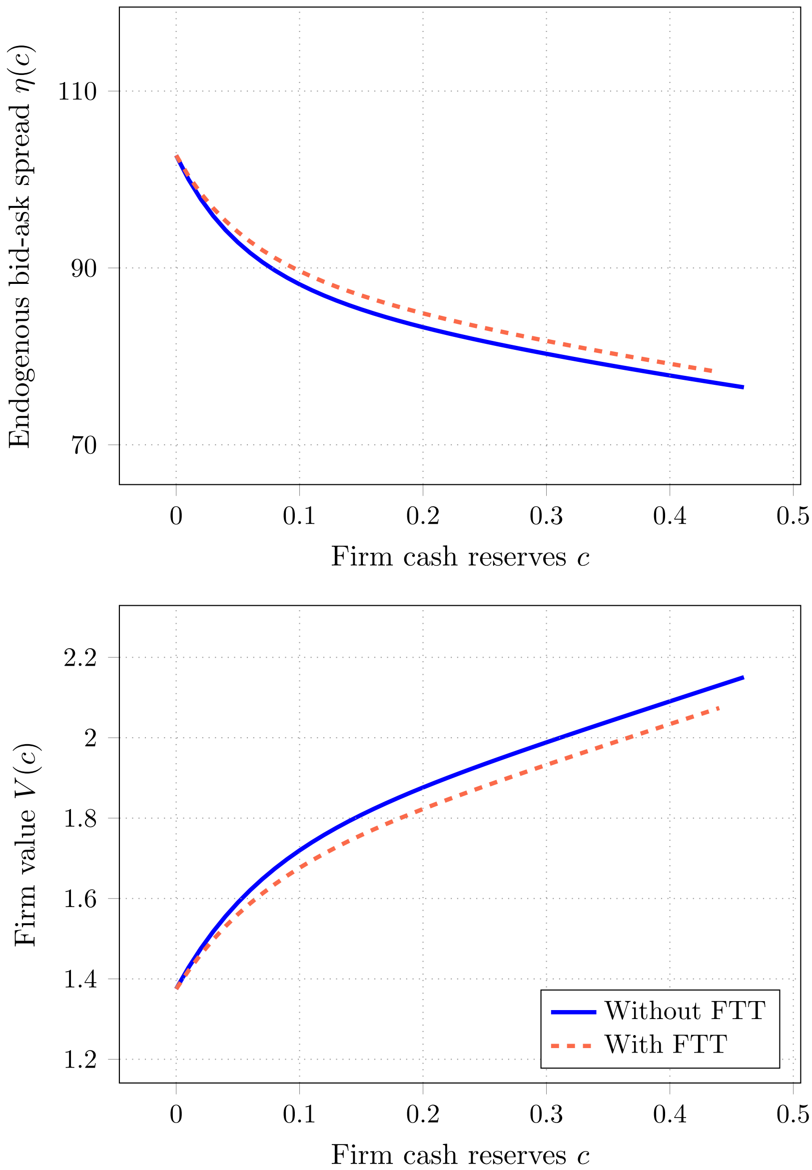
The figure shows the probability of liquidation (first panel), the probability of external financing (middle panel), and the probability of payout (bottom panel) as a function of the participation cost γ (left panel) and the funding cost κ (right panel) borne by liquidity providers. The blue solid line refer to the benchmark environment in which shareholders face no bid-ask spread when trading the stock, whereas the red dashed lines refer to the case in which the bid-ask spread is endogenously set by liquidity providers.

Figure 6: Firm-funded designated market makers (DMM).



The figure shows the endogenous bid-ask spread (in basis points) as well as firm value as a function of the firm cash reserves c. The solid blue line depicts the case in which the firm does not enter the contract with the designated market maker (DMM), whereas the dashed red line depicts the environment in which the firm enters the DMM contract.

Figure 7: Financial transaction tax (FTT)



The figure shows the endogenous bid ask-spread (in basis points) as well as firm value as a function of the firm cash reserves c. The solid blue line depicts the environment with no financial transaction tax (FTT), whereas the dashed red line depicts the environment in which investors face a FTT.