Reinforcing constraints *

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July 22, 2019

Abstract

Small firms face financing frictions, and trading their stocks entails non-negligible bid-ask spreads. I develop a model that studies how these characteristics are related and investigates their real effects. Bid-ask spreads increase both the firm's cost of equity and the opportunity cost of cash, which lead to tighter financial constraints, higher liquidation risk, and underinvestment. The ensuing decline in firm value affects the participation of competitive liquidity providers and widens the bid-ask spread further, then amplifying its detrimental effects on corporate policies and value. The model shows how frictions affecting liquidity providers are passed on to small firms (and their investors) and investigates some regulatory proposal affecting financial markets from a corporate perspective.

Keywords: Financial constraints, transaction costs, real effects of financial markets, small firms

JEL Classification Numbers: G32; G35; G31; G28

*I thank Erwan Morellec for many insightful conversations and Pierre Collin-Dufresne, Sebastian Gryglewicz, Semyon Malamud, and Boris Nikolov for valuable advice. I also thank Jack Bao, Isabella Blengini, Anna Cieslak, Stefano Colonnello, Marco Della Seta, Michael Fishman, Luigi Guiso, Giang Hoang, Julien Hugonnier, Dalida Kadyrzhanova, Nataliya Klimenko, Arvind Krishnamurthy, Antonio Mello, Konstantin Milbradt, Kjell Nyborg, Dimitris Papanikolaou, Sebastian Pfeil, Paola Sapienza, Costis Skiadas, Philip Valta, Zexi Wang, Peng Zhun, and the seminar participants at the Bank of Italy, Caltech, Copenhagen Business School, EPFL/University of Lausanne, the Federal Reserve Board, HEC Paris, Johns Hopkins University, Northwestern University, KU Leuven, the London School of Economics, the Stockholm School of Economics, the Toulouse School of Economics, the University of Wisconsin - Madison, the University of Zurich, and WU Vienna. Any remaining errors are my own. The views expressed are those of the author and should not be interpreted as reflecting the views of the Federal Reserve System or its staff.

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1 Introduction

Despite their humble market capitalization, small and micro firms represent more than 80% of U.S. firms over the past forty years (see, for instance, Hou, Xue, and Zhang, Forthcoming). Two key attributes characterize these firms. First, small firms are largely financially constrained. Small firms often face delays and costs when raising fresh funds from primary markets, an issue that has spurred the creation of an ad-hoc committee within the U.S. Securities and Exchange Commission (SEC).[1] Second, their stocks are costly to trade on secondary markets because of larger bid-ask spreads, lower trading volume, and other microstructure frictions (see, e.g., Hou, Kim, and Werner, 2016; Novy-Marx and Velikov, 2016; Chordia, Roll, and Subrahmanyam, 2011).

This paper develops a dynamic model that studies the intertwined relation of these two characteristics. The model focuses on a small firm that has assets in place, which generate a stochastic flow of revenues, and a growth option. As in the real world, the small firm faces financing frictions. Following Hugonnier, Malamud, and Morellec (2015) and Bolton, Chen, and Wang (2013), financing frictions are modeled by assuming that the firm faces uncertainty regarding its ability to raise fresh funds. The key departure from previous dynamic corporate finance models with financing frictions is the explicit consideration of costs and losses faced by shareholders and liquidity providers when trading the firm stocks. The firm maximizes its value by deciding how much cash to retain, pay out, raise from capital markets, and whether to invest in the growth option. Importantly, the model illustrates how corporate policies and value reflect as well as affect the cost of trading the stock.

To disentangle the forces at play, I start by examining an environment in which the bid-ask spread associated with the firm's stock is constant over time. To compensate for this

[1]Small firms are typically less known and more vulnerable to capital market imperfections. In contrast, large, established firms are more likely to have continued relations with financial institution and are less subject to asymmetric information. The U.S. SEC Advisory Committee in Small and Emerging Companies pointed out that small firms often struggle to attract capital, see https://www.sec.gov/spotlight/advisory-committee-on-small-and-emerging-companies.shtml.

cost, the return required by the investors (and, thus, the cost of equity) increases, more so if the bid-ask spread is wider. This greater return required by investors depresses the stock price and, thus, makes outside financing more costly. All else equal, this effect makes cash reserves more valuable and, thus, should incentivize the firm to keep more cash. At the same time, however, the greater return required by the investors also generates an offsetting strength: it expands the wedge between the firm's cost of capital and the return on cash, then increasing the opportunity cost of cash (and, thus, of retained earnings). On net, I show that firms whose stocks are traded at larger bid-ask spreads are more financially constrained, because both external and internal financing is more costly. As a result, these firms are more likely to face forced liquidations. Moreover, these firms face a severe underinvestment problem, as the additional return required by the investors erodes the profitability of investment opportunities. Overall, all these effects decrease firm value.

I then relax the assumption of constant, exogenous bid-ask spread. To do so, I put more structure into the model and realistically assume that liquidity providers in the market for the stock are perfectly competitive and face participation frictions (for instance, participation fees or funding constraints). Larger frictions faced by liquidity providers lead, all else equal, to larger bid-ask spread, consistent with the empirical evidence in Comerton-Forde et al. (2010) and Aragon and Strahan (2012), among others. As shown in the first part of the model, larger bid-ask spreads reduce firm value. When liquidity provision is endogenous, such a decrease in firm value feeds back into the bid-ask spread. In particular, liquidity providers need to extract larger rents from shocked shareholders as a proportion of the value of their claim to cover their participation fee. As a result, the bid-ask spread widens. Thus, frictions and costs faced by liquidity providers are passed on to the small firm's investors and, through this channel, affect corporate outcomes—exacerbating financial constraints, triggering underinvestment, and increasing the probability of forced liquidations. Thus, the paper provides a channel through which small firms' financing frictions and stock illiquidity reinforce each other.

The model delivers a rich set of testable predictions.[2] First, it characterizes corporate policies and outcomes for those firms (such as small and micro ones) whose stocks are costly to trade because of larger bid-ask spreads and other microstructure frictions. The model predicts that these firms should face severe financial constraints (because of their larger costs of external and internal financing), higher liquidation risk, and greater underinvestment problems. Notwithstanding these constraints, these firms should display larger payout rates in the cross-section to compensate for the larger investors' cost of trading, consistent with the evidence in Banerjee, Gatchev, and Spindt (2007). Moreover, the model can rationalize the evidence that small firms' bid-ask spreads increase much more in the cross-section when liquidity providers' funding constrants tighten, as illustrated by Anand et al. (2013) and Aragon and Strahan (2012). In fact, these participation frictions affect the bid-ask spread both by making liquidity provision more costly and by deteriorating firm value.[3]

Furthermore, the model provides a tractable framework to evaluate regulatory proposals targeting equity markets (and their investors) from a corporate perspective. First, academics and policy-makers have been recently questioning whether market forces guarantee enough liquidity provision, especially in the market for smaller exchange-listed companies. Several stock markets have then contemplated the possibility for firms to engage a designated market maker (DMM) to maintain the bid-ask spreads below an agreed-upon level. While DMMs can be effective at decreasing the firm's cost of capital by capping investors' trading costs, they bind the firm to correspond a cash flows to the DMM. On net, the model predicts that small, financially constrained firms would hardly find this contractual agreement value-enhancing. Furthermore, the model can also help understand the effects of financial transaction taxes (FTTs) on corporations. Through the mechanism described in the paper, FTTs increase the cost of trading borne by in-

[2]Section 5 confirms the robustness of these predictions when introducing bank credit in the model as an additional source of financing. The analysis abstracts from corporate bonds as small firms find this source of financing too costly or unfeasible.

[3]The second channel should be less relevant for large firms, characterized by easy access to several sources of outside financing, and whose stocks are traded relatively more often and at small bid-ask spreads.

vestors and, thus, exacerbate the issuing firm's financial constraints, fuel liquidation risk, depress investment, and lower firm value. FTTs would be particularly harmful for small stocks, as these firms already suffer from larger bid-ask spreads and other microstructure frictions. The model then suggests that policies making the FTT contingent on a firm's market capitalization could relieve these harmful effects.[4]

Related literature Although small and micro firms represent a staggering fraction of the number of U.S. listed firms, corporate finance models are hardly designed for these firms. Indeed, while there are many studies focusing on startups or growth firms in tech industried that are not yet public, there are very few studies that explicitly focus on small public firms (see, e.g., Mehran and Peristiani, 2010). To the best of my knowledge, this paper is the first to systematically study the real, intertwined effects of two market imperfections that grip these firms: financing frictions faced by the firm when raising fresh funds and frictions faced by their shareholders when trading the stock. By showing that the interaction of these frictions increases their liquidation probability, this paper provides a rationale as to why small firms are fading from exchanges, as their propensity to list has fallen sharply and their delisting rate has increased in the last decades, see Kahle and Stulz (2017) and Doidge, Karolyi, and Stulz (2017).

A growing literature shows that frictions affecting stock trading impact corporate policies and outcomes. Fang, Noe, and Tice (2009) show that firms with liquid stocks are more valuable. Campello, Ribas, and Wang (2014) find that stock liquidity improves corporate investment and value. Similarly, Amihud and Levi (2018) show that corporate investment declines with stock illiquidity. Banerjee, Gatchev, and Spindt (2007) suggest that firms with illiquid stocks pay out more dividends. Nyborg and Wang (2019) find a positive causal relation of stock liquidity on corporate cash holdings, which is robust to different measures of liquidity. Brogaard, Li, and Xia (2017) find that stock market liquidity reduces firms' bankruptcy risk. This paper provides a unified theoretical frame-

[4]That said, the analysis is silent on the desirability (the welfare gains) arising from this tax, in that the focus of the paper is on the effects on the corporate sector.

work that can explain these findings. In addition, it discusses the corporate implications of some regulatory proposals aimed at targeting financial markets.

Since the 2007-2009 financial crisis, there has been a large interest in understanding the effects of liquidity providers' funding liquidity on the market liquidity of securities. Many empirical works show that frictions faced by liquidity providers have an important impact on the stocks they trade; see, for instance, Anand et al. (2013); Hameed, Kang, and Viswanathan (2010); Aragon and Strahan (2012); Comerton-Forde et al. (2010). In the meanwhile, a theoretical literature has developed studying the dynamics of liquidity (demand and supply) and their impact on asset prices; see, for instance, Budish, Cramton, and Shim (2015) or Huang and Wang (2010, 2009). These models abstract away from informational frictions but, rather, focus on financial constraints and margin requirements faced by market participants (like intermediaries, broker-dealers, and other liquidity providers). A key assumption in these models is that the security traded by market participants promises a constant (exogenous) flow of dividends. In contrast, the current paper tries to endogenize the dividend flow associated with the traded security by investigating the optimal corporate policies of the issuing firm.

The influential work by Brunnermeier and Petersen (2008) shows that there is a two-ways link between an asset's market liquidity and traders' funding liquidity. Traders provide market liquidity, which in turn depends on their funding ability. Because of margin requirements, traders' funding depends on the assets' market liquidity. The current paper instead focuses on the relation between the funding liquidity of a given (nonfinancial) small/micro firm and the market liquidity of its stocks. It shows that firm financing frictions and stock illiquidity reinforce each other, then providing a rationale as to why the degree of financial constraints and stock market illiquidity are disproportionately greater for small/micro stocks vis-à-vis large firms.

Finally, this paper contributes to the strand of dynamic corporate finance models with financing frictions, including Décamps et al. (2011, henceforth DMRV); Bolton, Chen, and Wang (2011, henceforth BCW); Hugonnier, Malamud, and Morellec (2015, henceforth HMM); Malamud and Zucchi (2019); or Della Seta, Morellec, and Zucchi (Forthcoming). These papers show that financing frictions, such as costs or uncertainty in raising external funds, should increase a firm's propensity to keep precautionary reserves. While these extant papers impose an exogenous cost of holding cash, the current model shows that this cost can arise endogenously when accounting for trading frictions faced by firm shareholders. In particular, I show that trading frictions impact both the cost of internal and external financing.

The paper proceeds as follows. Section 2 describes the baseline model. Section 3 analyzes the effects on corporate policies of a constant bid-ask spread. Section 4 endogenizes liquidity provision in the market of the stock and, thus, the bid-ask spread, and studies some regulatory proposals targeting financial equity markets from a corporate perspective. Section 5 assess the robustness of the results. Section 6 concludes. Proofs are gathered in the Appendix.

2 The model

Time is continuous, and uncertainty is modeled by a probability space $(\Omega, \mathcal{F}, P)$ equipped with a filtration $(\mathcal{F}_t)_{t\geq 0}$. Agents are risk-neutral and discount cash flows at a constant rate $\rho > 0$.

The firm I consider a small firm operating a set of assets in place, which generate a continuous and stochastic flow of revenues. The flow of revenues is modeled as an arithmetic Brownian motion, $(Y_t)_{t\geq 0}$, whose dynamics evolve as

$$dY_t = \mu dt + \sigma dZ_t. \tag{1}$$

The parameters μ and σ are strictly positive and represent the mean and volatility of corporate revenues, and $(Z_t)_{t\geq 0}$ is a standard Brownian motion. The firm has access to a growth option that has the potential to increase its income stream from dY_t to

$dY_t^+ = dY_t + (\mu_+ - \mu)dt,\ \mu_+ > \mu$, by paying a lump-sum cost $I > 0$. That is, the cash flow drift can assume two values $\mu_i = \{\mu, \mu_+\}$. Investment is assumed to be irreversible.

The cash flow process in equation (1) implies that the firm can make operating profits and losses. If capital supply was perfectly elastic, operating shortfalls could be covered by raising outside financing immediately and at no cost. In practice, small firms face financing frictions, such as uncertainty or costs in their ability to raise funds. I model capital supply uncertainty as in HMM and assume that the firm raises new funds at the jump times of a Poisson process, $(N_t)_{t\geq 0}$, with intensity λ. That is, if the firm decides to raise outside funds, the expected financing lag is $1/\lambda$ periods. When $\lambda \to 0$, the firm cannot raise external funds at all (equivalently, it takes an infinite waiting period to raise fresh funds upon searching) and relies on cash reserves to cover unexpected operating losses. When $\lambda \to \infty$, conversely, the waiting time upon searching for external funds is zero—i.e., the firm has access to outside financing whenever needed with no delays. Notably, as shown in the paper, the discount on newly-issued equity is related to secondary market frictions.

Because the firm faces financing frictions, it has incentives to retain earnings in cash reserves. I denote by $(C_t)_{t\geq 0}$ the firm's cash reserves at any $t \geq 0$. Cash reserves earn a constant rate, $r \leq \rho$. Whenever $r < \rho$, keeping cash entails an opportunity cost.[5] In contrast with extant cash holdings models—in which the strict inequality $r < \rho$ is needed to depart from the corner solution featuring firms piling infinite cash reserves—I allow for the $r = \rho$ case. The cash reserves process satisfies:[6]

$$dC_t = rC_t dt + \mu_i dt + \sigma dZ_t - dD_t + f_t dN_t. \tag{2}$$

In this equation, $dD_t \geq 0$ represents the instantaneous flow of payouts at time t. The

[5] This cost can be interpreted as a free cash flow problem à la Jensen (1986) or as tax disadvantages (see Graham, 2000).

[6] When investment occurs (meaning that the cash flow drift goes from μ to μ_+), the cost I is financed either with cash or external financing. Because the paper focuses on the decision of whether or not to invest (rather than on the investment timing), I do not explicitly model the outflow I when the growth option is exercised (which can also be financed with outside funds).

process $(D_t)_{t\geq 0}$ is non-decreasing, reflecting shareholders' limited liability. $f_t \geq 0$ denotes the instantaneous inflow of funds when financing opportunities arise, in which case management stores the proceeds in the cash reserves. This assumption is consistent with the strong, positive correlation between equity issues and cash accumulation documented by McLean (2011) or Eisfeldt and Muir (2016). The cash reserves increase with external financing, retained earnings, and the interest earned on cash, whereas they decrease with payouts and operating losses. The controls D and f are endogenously characterized. Notably, the level of the firm's cash reserves can be seen as a gauge of the firm's financial strength—i.e., how much internally-generated or outside funding is readily available.

Management can distribute cash and liquidate the firm's assets at any time. Yet, liquidation is inefficient, as the recovery value of assets in liquidation, denoted by ℓ, is smaller than the firm's first best, μ/ρ. These costs erode a fraction, $1-\phi \in (0,1]$, of the firm's first best, so the liquidation value is $\ell = \phi\mu_i/\rho$. We denote by τ the endogenous default time of the firm.

Transacting the firm stocks The key departure from previous dynamic corporate finance models with financing frictions is the explicit consideration of secondary market stock transactions. There are two types of risk-neutral traders: investors (who may buy, hold, and eventually sell the stock) and trading firms (or liquidity providers, which ease investors' trading). As in Budish, Cramton, and Shim (2015) or Huang and Wang (2010), the model abstracts from asymmetric information about the current value of the firm.

Investors are ex-ante identical and infinitely lived. Each of them has measure zero and cannot short sell. Investors can be hit by liquidity shocks. Following previous contributions (e.g., Bessembinder, Hao, and Zheng, 2015), liquidity shocks lead to a sudden need for liquidity that reduces the subjective valuation of the asset by a fraction χ.[7] Thus, χ can be interpreted as the opportunity cost of being locked into an undesired asset position—for instance, because of take-it-or-leave-it investment opportunities, un-

[7]As in Bessembinder, Hao, and Zheng (2015), the cost of liquidity shocks is proportional to the fundamental value of the asset held by the investor.

predictable financing needs, or unpredictable changes in hedging needs. The liquidity shock vanishes once the shocked investor sells his stock or he bears the loss χ. Liquidity shocks are independent across investors and occur at the jump times of a Poisson process with intensity $\delta > 0$. Conversely, non-liquidity-shocked shareholders have no immediate need to trade and, thus, are indifferent between keeping the stock or selling it at its fundamental value. The mass of non-liquidity-shocked investors is larger than that of liquidity-shocked shareholders.

Trading firms are agents who maintain an active presence and provide liquidity in the market of the stock.[8] They have no intrinsic demand to buy or sell the firm's assets and are not subject to liquidity shocks. Trading firms compete à la Bertrand in providing liquidity to shocked investors. Trading firms pay a fixed flow cost as long as they are active in the market of the stock, denoted by γ, which can be interpreted as the cost of monitoring and processing market movements. We assume that liquidity providers are financially constrained. Their cost of funding erodes a fraction κ of their gross gain from liquidity provision.

Trading firms post bid and ask quotes. On the ask side, they trade with investors who are not liquidity-shocked (i.e., do not have an immediate need to trade), who are indifferent between staying out of the market or buying the stock at its fundamental value. As a result, the gain to trading firms on this side of the transaction is null. On the bid side, trading firms trade with shocked shareholders. Because shocked shareholders value the asset at a discount χ, trading firms can extract surplus from this side of the transaction. Trading firms' gain from transacting with shocked shareholders is denoted by $\eta \leq \chi$. The quantity η then represents the difference between the ask and the bid price (i.e., the bid-ask spread). This quantity is assumed as exogenous in Section 3 and endogenized in Section 4.

[8]Trading firms can be interpreted as high frequency traders, market makers, or algorithmic traders as in Budish, Cramton, and Shim (2015), or simply as agents maintaining a constance market presence such as trading desks and hedge funds, as in Huang and Wang (2010). For simplicity, the size of the trading firm sector is normalized to one.

Equilibrium Corporate decisions—cash retention and payout (D), financing (f), liquidation (τ), and investment (μ_i)—are set to maximize the following expression:

$$V(c) = \sup_{(D,f,\tau,\mu_i)} \mathbb{E}\left[\int_0^\tau e^{-\rho t}\left(dD_t - f_t\, dN_t - dB_t\right) + e^{-\rho\tau}\ell\right], \tag{3}$$

which represents the expected present value of net payouts (the first term) plus the liquidation value of assets (the second term). Notably, the expected flow of payouts to shareholders is drained by the losses they bear when trading (whose cumulative process is denoted by B_t). Payouts and trading costs are endogenously determined. In equilibrium, the bid-ask spread leaves trading firms indifferent between providing liquidity in the market of the stock and staying out of the market. Via the trading firms' decision to provide liquidity, the bid-ask spread not only affects, but also reflects corporate policies and value.

2.1 Discussion of the assumptions

The two key features of the model are: (a) frictions faced by the firm's shareholders when trading the stock, (b) frictions faced by the firm as a whole when raising fresh financing. While the model can apply to any firm subject to these two frictions to a certain degree, it is especially relevant for small and micro firms, which are fraught with financing frictions and whose stocks are traded at non-negligible bid-ask spreads.[9] In modeling frictions (a)-(b), the paper takes a parsimonious approach, in an attempt to nest insights coming from secondary market trading into a dynamic corporate finance model with financing frictions.

More specifically, the modeling of secondary market transactions is flexible enough to apply to small stocks traded on major exchanges as well as stocks traded in over-

[9]Large firms have an easier access to several sources of financing (like corporate bonds or commercial paper), and their stocks are traded at tiny bid-ask spreads. Whereas large firms dwarf small firms capitalization-wise, the number of small and micro firms exceeds by far that of large firms, representing more than 80% of the number of listed firms.

the-counter markets. One key difference of this paper vis-à-vis models of the effects of liquidity demand/supply on asset valuations is the explicit focus on the policies of the issuing firm (rather than on a full-fledged description of secondary market transaction). Whereas previous contributions usually take the flow of dividends associated with a given stock as constant, the current paper endogenizes it.

To keep the analysis tractable, two assumptions are made. First, the trading costs borne by shocked (selling) investors are positive, whereas the costs borne by (buying) non-shocked investors are zero. This assumption is consistent with Brennan et al. (2012), who show that sell-order frictions are priced more strongly than buy-order ones.[10] Second, in modeling trading frictions, this paper focuses on participation and funding frictions rather than on informational ones. Our focus is motivated by recent empirical evidence (see, for instance Chung and Huh, 2016) showing the important impact of the non-information cost of trading on asset prices. This paper builds on this strand and seeks to focus on the implications for corporate outcomes.

Turning to firm financing frictions, the model assumptions regarding the firm's financing menu aim at keeping the setup as parsimonious as possible. Following Hennessy and Whited (2007), BCW, or HMM, I do not micro-found the nature of financing frictions and model any adverse selection or limited commitment problem in a reduced form fashion. Financing frictions are modeled as uncertainty in the firm's ability to raise fresh funds, an issue that is especially severe for small firms, as also pointed out by the U.S. SEC Advisory Committee in Small and Emerging Companies. As illustrated by the survey of Lins, Servaes, and Tufano (2010), financing uncertainty is one of the top reasons behind corporate cash stockpiling. Thus, the model allows firm management to pile earnings inside a cash reserves. The magnitude of the cash reserves is a gauge of the firm's financial strength (i.e., of the firm's ability to withstand operating losses).

Finally, it is worth noting that small/micro firms find it too costly (or unfeasible) to ac-

[10] Brennan et al. (2012) show that the pricing of illiquidity emanates principally from the sell-side. The idea is that agents seldom face needs to buy stock urgently, but unexpected needs for cash may force them to sell stock suddenly.

cess corporate bond financing. Rather, these firms usually access debt by borrowing from banks—for instance, by securing themselves credit line availability. While the baseline version of the paper abstracts from this source of financing (for the sake of tractability), credit line availability is introduced in Section 5.2. The model predictions are robust to this model extension.

3 The effect of bid-ask spreads on corporate policies

To disentangle the economic effects at play, it is useful to start by analyzing the firm's decision-making problem when the firm's shareholders face an exogenous, fixed bid-ask spread η. This assumption will be relaxed in Section 4 (in which η and firm value are jointly, endogenously determined).

3.1 Deriving firm value

Because liquidity shocks are independent across investors, a measure δdt of shareholders is shocked on each time interval. Shareholders seek to sell the stock as soon as hit by a liquidity shock. When trading with shocked shareholders, trading firms capture a fraction η of the surplus created, which means that the transaction price of the aggregate claim of shocked shareholders is: $\delta\Phi(c) \equiv (1-\eta)\delta V(c)$. As long as $\delta\Phi(c) > (1-\chi)\delta V(c)$ (which is the case if $\eta < \chi$), liquidity-shocked shareholders are better off selling the assets (entailing the loss η) than keeping it (entailing the loss χ). The quantity $\delta[\Phi(c) - V(c)] = -\delta\eta V(c)$ represents the associated (aggregate) loss to shocked shareholders.

I next show that this loss affects corporate policies and firm value. Assume first that the firm does not have growth options[11] and consider retention and payout policies. As in previous cash management models, the benefit of holding cash is decreasing in cash reserves. The (opportunity) cost is the wedge between the return required by the investors

[11] Solving for firm value when there are no growth option is auxiliary to studying the optimal investment rule, as in HMM. The optimal investment rule is reported in Proposition 4.

and the return on cash. I conjecture (and verify) that there is a target cash level, C_V, at which the cost and benefit of cash are equalized. Above C_V, it is optimal to pay excess cash out. Below C_V, shareholders retain earnings in cash reserves and search for financing. When operating losses cannot be covered by drawing funds from cash reserves or by raising fresh equity, the firm is forced into liquidation. The time of liquidation is then the first time the cash reserves process hits zero:

$$\tau = \inf\left\{t \geq 0 : C_t \leq 0\right\}. \tag{4}$$

By Itô's lemma, equity value satisfies the following ordinary differential equation (ODE) for any $c < C_V$:

$$\rho V(c) = (rc + \mu) V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda \sup_f [V(c+f) - f - V(c)] + \delta [\Phi(c) - V(c)]. \tag{5}$$

The left-hand side is the return required by the investors. The first two terms on the right-hand side represent the effect of cash retention and cash flow volatility on equity value. The third term represents the surplus from raising external financing, i.e., the probability-weighted surplus accruing to incumbent shareholders. In the appendix, $V(c) - c$ is shown to increase with c, so it is optimal to raise the cash buffer up to C_V whenever financing opportunities arise.[12] Thus, the optimal refinancing amount is $f(c, \Phi) = C_V(\Phi) - c$. The last term reflects the loss borne by liquidity-shocked investors when selling the firm's stocks. Substituting $\Phi(c)$ and $f(c, \Phi)$ into equation (5) gives

$$(\rho + \delta\eta) V(c) = (rc + \mu) V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda [V(C_V) - V(c) - C_V + c]. \tag{6}$$

The left-hand side reveals that the bid-ask spread increases the return required by investors by $\delta\eta \geq 0$. This additional compensation leads to an increase in the opportunity cost of cash from $\rho - r$ to $\rho + \delta\eta - r$. Equation (6) is solved subject to the following

[12] The marginal value of cash is greater than one, i.e. $V'(c) \geq 1$ (see Lemma 6 in Appendix A for a proof). This implies that the first derivative of $V(c) - c$ is non-negative.

boundary conditions. The firm is liquidated when cash is exhausted and the firm cannot raise new funds. Thus, the condition

$$V(0) = \ell \tag{7}$$

holds when the firm runs out of cash. Moreover, it is optimal to distribute cash exceeding C_V with dividends or share repurchases.[13] Firm value is thus linear above C_V: $V(c) = V(C_V) + c - C_V$ for any $c \geq C_V$. Subtracting $V(c)$ from both sides of this equation, dividing by $c - C_V$, and taking the limit as c tends to C_V gives

$$\lim_{c \uparrow C_V} V'(c) = 1. \tag{8}$$

That is, it is optimal to start paying out cash when the marginal value of one dollar inside the firm equals the value of a dollar paid out to shareholders. The target cash level that maximizes shareholder value is determined by the super-contact condition,

$$\lim_{c \uparrow C_V} V''(c) = 0. \tag{9}$$

Both the target cash level C_V and the issue size f depend on η. Below I better investigate corporate policies vis-à-vis an environment in which the bid-ask spread is zero.

Cash management and payouts. In the presence of financing frictions, the benefit of cash stems from guaranteeing financial flexibility to the firm. This benefit decreases with cash reserves. A firm's target cash level balances this benefits with the opportunity cost of holding cash. Previous cash management models take the cost as exogenous and constant. In this paper, differently, bid-ask spreads generate a wedge between the return on cash and the return required by the investors. This model then delivers finite target cash levels even when r and ρ coincide.[14]

[13] As shown in Section 5.1, it is never optimal to buy back the shares of shocked investors when $c < C_V$.

[14] In previous dynamic cash management models, holding cash is not costly if r and ρ coincide and, thus, a financially constrained firm would pile infinite cash reserves.

Consider first the extreme case in which financing frictions are so severe that the firm has no access to external financing (i.e., $\lambda = 0$). The bid-ask spread does not affect firm's refinancing decisions in this case (as the firm has no access to outside financing) and, thus, does not directly affect the benefit of cash reserves. However, bid-ask spreads increase the cost of holding cash by increasing the wedge between the return on cash and the return required by the investors. By increasing the cost of cash, the bid-ask spread leads to a decrease in the target cash level.

When the firm has access to external financing ($\lambda > 0$), the bid-ask spread has two distinct, offsetting effects on the incentive to keep cash. First, it increases the cost of cash through the mechanism explained above. This effect leads to a reduction in the target cash level. Second, the bid-ask spread depresses the stock price and, for a given issue size, it reduces the surplus from outside financing. *Ceteris paribus*, this effect makes cash reserves more valuable and, thus, generates an incentive to increase the target cash level. The next proposition shows that the first effect dominates. I denote by C^* the target cash level when trading the firm's stock is frictionless.

Proposition 1 (Cash management) *All else equal, the bid-ask spread leads to a decrease in the target cash level on net; i.e., the inequality $C_V \leq C^*$ holds. The target level C_V decreases with the magnitude of the bid-ask spread η.*

Proposition 1 illustrates that the first effect dominates as the target cash level decreases with the bid-ask spread (see Appendix A.1 for a proof). This model prediction is consistent with Nyborg and Wang (2019) and Bakke, Jens, and Whited (2012), who find that less liquid stocks hold smaller cash reserves.

Cash retention and payout decisions are closely intertwined. I define the payout probability as follows:

$$P^p(c, C_V) = E_c\left[e^{-\lambda \tau_d(C_V)}\right],$$

where $\tau_d(C_V)$ represents the first time that the cash reserves process, initially at c, first reaches the payout threshold C_V. The firm's bid-ask spread enters these probabilities through $\tau(C_V)$—i.e., by affecting the target cash level, the bid-ask spread affects the payout frequency. The following proposition studies these probabilities.

Proposition 2 (Payout probability) *Trading costs lead to an increase in the probability of payout,* $P^p(c, C_V) > P^p(c, C^*)$.

Proposition 2 implies that a firm with costly stocks pays out more dividends. In so doing, it compensates ex ante the frictions that shareholders bear when trading the stock. This finding is in line with Banerjee, Gatchev, and Spindt (2007), who advance the idea that investors view stock market liquidity and dividends as substitutes. When a firm's bid-ask spread is small, investors can create dividends to themselves by cashing out their investment. When the bid-ask spread is large, investors require the firm to pay out more dividends.

One question may arise as to why the firm does not commit to a payout flow (either dividends or share repurchases) even when the cash reserves are below the target level too. Section 5.1 shows analytically that this policy would be suboptimal, as cash would be paid out to shareholders when its marginal value is greater than one (meaning that cash is more valuable inside than outside the firm).

Liquidation and financing. Because the bid-ask spread increases the cost of internal financing (by increasing the opportunity cost of cash) and external financing (by increasing the return required by the investors), a question arises as to how the firm's financial resilience is affected. To do so, I study the firm probability of liquidation and financing. I define the probability of liquidation while the firm is looking for external funds as

$$P^l(c, C_V) = E_c \left[e^{-\lambda\tau(C_V)} \right]$$

and, complementarily, the probability of external financing: $P^f(c, C_V) = E_c \left[1 - e^{-\lambda\tau(C_V)} \right]$. The bid-ask spread enters these probabilities through $\tau(C_V)$, representing the first time that the cash process, reflected from above at C_V, is absorbed at zero. The following proposition studies these probabilities.

Proposition 3 (Probability of liquidation and financing) *The bid-ask spread increases the firm's probability of liquidation, $P^l(c, C_V) > P^l(c, C^*)$, and decreases the probability of external financing, $P^f(c, C_V) < P^f(c, C^*)$.*

Taken together, these results suggest that bid-ask spreads exacerbate firms' financial constraints through different, albeit interrelated, channels. First, they increase the cost of internal financing, making the firm less willing to keep cash reserves. Second, bid-ask spreads increase the cost of external financing and reduce the firm's probability of issuing fresh equity, because the discount due to secondary market frictions reduces the surplus accruing to current shareholders. Overall, bid-ask spreads increase the probability of liquidation, consistent with Brogaard, Li, and Xia (2017).

Investment decisions So far, I focused on firm value for a given value of μ_i (i.e., the value of the firm with no investment opportunities). Following Décamps and Villeneuve (2007) and HMM, the value of the firm with no investment opportunities serves to derive the zero-NPV cost—i.e., the maximum amount that the firm is willing to pay in order to exercise the growth option (see Appendix A.4 for a proof). The next proposition illustrates how the bid-ask spread affects such zero-NPV cost.

Proposition 4 *The zero-NPV cost is given by*

$$I_V = \frac{\mu_+ - \mu}{\rho + \delta\eta} - (C_{V+} - C_V)\left[1 - \frac{r}{\rho + \delta\eta}\right], \tag{10}$$

where C_{V+} denotes the target cash level after the growth option is exercised.

If, instead, the bid-ask spread was zero, the zero-NPV cost would be

$$I^* = \frac{\mu_+ - \mu}{\rho} - (C_+^* - C^*)\left(1 - \frac{r}{\rho}\right), \tag{11}$$

with C_+^* denoting the post-investment target cash level. Comparing (10) and (11) reveals that the bid-ask spread leads to a decrease in the investment reservation price (i.e., $I_V < I^*$), i.e., it reduces the maximum amount that the firm is willing to pay to invest in the growth option. If the investment cost lies in the interval $[I_V, I^*]$, the growth option has negative NPV if the bid-ask spread is positive $\eta > 0$, whereas it has positive NPV if the bid-ask spread is zero. That is, the underinvestment problem worsens if the bid-ask spread widens, a result that is empirically supported by Amihud and Levi (2018). The severity of the underinvestment problem can be approximated as follows:[15]

$$\Delta I_V = I_V - I^* \approx -\frac{\mu^+ - \mu}{\rho} \frac{\delta\eta}{\rho + \delta\eta} < 0. \tag{12}$$

Notably, the gap between I_V and I^* increases with η; that is, the underinvestment problem is more severe when transacting the security is more costly.

Quantitative analysis This section provides a quantitative assessment of the model predictions so far, i.e., when the firm bid-ask spread is exogenous. Table 1 reports the baseline parameterization.

Insert Table 1 Here

The risk-free rate ρ is set to 2%, and the return on cash is set to 1%. The resulting opportunity cost of cash is equal to 1%, as in BCW and DMRV. Small firms tend to have lower profitability in the cross section (see, e.g., Fama and French, 2008). The cash flow drift μ is therefore set to 0.05, which is lower than the value used by DMRV and consistent with the bottom range of values in Whited and Wu (2006). Upon exercising the growth option, the cash flow drift is assumed to be 20% bigger (i.e., $\mu_+ = 0.06$). Turning to cash flow volatility, I set $\sigma = 0.12$, which is consistent with Graham, Leary, and Roberts (2015) and is higher than the value set by DMRV (as small firms have more volatile cash flows). I base the value of liquidation costs on the estimates of Glover (2016)

[15] The difference between target cash levels (the second term in the expressions of I_V and I^*) plays a second-order effect, as in the model of HMM.

and set $\phi = 0.55$. The parameter λ is set to 0.75, which is consistent with Fama and French (2005) who report that roughly 73.7% of small firms issue new equity each year. The intensity of the liquidity shock is set to $\delta = 0.7$, as in He and Milbradt (2014). The magnitude of the bid-ask spread is varied extensively in the analysis in this section.[16]

Insert Table 2 Here

Table 2 illustrates the impact of bid-ask spreads of various magnitude on the target cash level, the probability of external financing, the probability of liquidation, the probability of payout, the investment reservation price (i.e., the zero-NPV cost), and firm value when cash reserves are at their target level. For a bid-ask spread equal to 60 basis points, the target cash level decreases by more than 13% with respect to the case in which the bid-ask spread is zero (i.e., the first line of the table). Because the bid-ask spread engenders a wedge between the return required by the investors and the return on cash, the firm pays out cash to investors more often. In fact, the table shows that the probability of payout increases on average by about 3.1% when the bid-ask is equal to 60 basis points.[17] The maximum investment threshold decreases by more than 17.8% vis-à-vis the environment in which the bid-ask spread is zero, engendering a notable underinvestment problem. Because both external and internal financing are more expensive in the presence of a positive bid-ask spread, the probability of liquidation increases by 2%, on average. Notably, the increase in the probability of liquidation is greater as cash reserves approach depletion (conversely, liquidation is less likely for large levels of cash reserves). Specifically, Table 3 illustrates how the probability of liquidation changes at different cash

[16]Chung and Zhang (2014) report the median bid-ask spread (calculated using TAQ data) for firms sorted by quintiles of market capitalization over the period 1993-2009. They report that the median bid-ask spread of smaller quintile firms is 0.0195 for NYSE/AMEX stocks and 0.0501 for NASDAQ stocks. They also note that the bid-ask spread has decreased over time (see also Hasbrouck, 2009): The median bid-ask spread for (all capitalization) NYSE/AMEX stocks went from 0.0094 in 1993 to 0.0034 in 2009, and from 0.0346 in 1993 to 0.0067 in 2009. In the model parameterization, I take a conservative approach and take a relatively low value for the bid-ask spread. In so doing, I show that even small bid-ask spread can bear substantial impact on corporate policies and value.

[17]To calculate these probabilities, I follow HMM and calculate the average for a cross-section of firms with cash reserves uniformly distributed between 0 and C_V.

levels when varying the magnitude of the bid-ask spread. For instance, when the firm stands at $C_V/4$, the probability of liquidation is equal to 14.1% if the bid-ask spread is zero, and equal to 18.3% if the bid-ask spread is equal to 60 basis points. It is also worth noting that, for a given cumulative shock, liquidation becomes relatively more likely if the bid-ask spread is larger. If the bid-ask spread was zero, a series of shocks reducing the cash buffer from $C^*/2$ to $C^*/4$ would increase the probability of liquidation from 1.97% to 14.1%. When the bid-ask spread is equal to 60 basis points, the reduction from $C_V/2$ to $C_V/4$ would be caused by a cumulative loss about 13% smaller (than in the case in which the bid-ask spread is zero), but it would increase the probability of liquidation from 3.35% to 18.3%. Overall, a bid-ask spread equal to 60 basis points leads to a more than 18% reduction in firm value if the firm holds its target cash level, at which level financial constraints are the loosest. Table 2 shows that such a loss as well as all the effects on corporate policies that we discussed above is sizable even for relatively smaller bid-ask spreads.

Insert Table 3 Here

To sum up, the analysis so far advances the following testable predictions. First, firms whose stocks have a larger bid-ask spread are more financially constrained, as the cost of external *and* internal financing is larger. As a result, the firm taps the equity market less often and keeps less cash reserves. Second, the firm is less resilient to negative operating shocks, and the probability of liquidation increases more steeply after a given cumulative shock. Third, the firm underinvests, as the additional return required by the investors erodes the profitability of investment opportunities and makes the firm forgo such opportunities (equivalently, it reduces the price at which the firm is willing to invest). Finally, firm value decreases substantially.

4 Endogenizing the bid-ask spread

Having analyzed how bid-ask spreads affect corporate financial policies, liquidation risk, investment, and firm value, I now derive the equilibrium bid-ask spread by assuming competition and participation frictions faced by liquidity providers. That is, in this section, η is endogenous, and affects as well as depends on firm value.[18] In the following, I interchangeably use trading firms or liquidity providers.

4.1 Endogenous liquidity provision and corporate policies

Trading firms are liquidity providers that are active on both the bid and ask sides of transactions, as in Section 3. The ask price is equal to the fundamental value of the firm, $V(c)$, as trading firms sell to non-liquidity-shocked investors on this side of the transaction. On the bid side, trading firms buy stocks from liquidity-shocked investors. Bertrand competition among trading firms implies that the equilibrium bid-ask spread is determined by the zero-profit condition, which is given by:

$$-\delta\,(1-\eta)\;V(c;\eta,C_V(\eta))+\delta V(c;\eta,C_V(\eta))(1-\kappa)=\gamma \quad s.t. \quad \eta\leq\chi \tag{13}$$

on any dt. The first term on the left-hand side is the price at which trading firms buy the stock from liquidity-shocked shareholders. The second term is the price at which trading firms sell the stock to non-shocked investors net of the funding cost. The right-hand side of this equation is the participation cost borne by trading firms to maintain constant market presence. Notably, the endogenous bid-ask spread set by trading firms affects *and* depends on firm value. Importantly, the equilibrium η cannot exceed χ, otherwise shocked shareholders would be better off holding the stock instead of selling it to trading

[18] A previous version of this paper focused on endogenizing the measure of active liquidity providers, yielding similar predictions (see Appendix B.1).

firms. Using standard arguments, firm value satisfies the following HJB equation:

$$\rho V(c;\eta) = (rc+\mu)\,V'(c;\eta) + \frac{\sigma^2}{2}V''(c;\eta) + \lambda \sup_f [V(c+f;\eta) - V(c;\eta) - f]$$
$$+\,\delta\left[(1-\min[\eta(c);\chi])V(c,\eta) - V(c,\eta)\right]. \tag{14}$$

The last term on the right-hand side is the loss borne by liquidity-shocked shareholders. The other terms in this equation admit an interpretation similar to equation (5) (i.e., as in the case in which the bid-ask spread is exogenous). Similar to the case with constant η, it is optimal to raise funds up to the target level whenever financing opportunities arise, i.e., $f = C_V(\eta) - c$. Using this result and condition (13), equation (14) can be re-written as follows:

$$(\rho+\delta\kappa)V(c) = (rc+\mu)\,V'(c) + \frac{\sigma^2}{2}V''(c) + \lambda\,[V(C_V) - C_V + c - V(c)] - \gamma \tag{15}$$

whenever $\eta(c) < \chi$. Equation (15) implies that the frictions borne by liquidity providers are passed on to the liquidity-shocked shareholders. As a result, frictions affecting the provision of liquidity end up affecting the value of the small firm. Equation (15) is solved subject to conditions similar to those in Section 3, i.e. $V(0) - \ell = \lim_{c\uparrow C_V} V'(c) - 1 = 0$ at the liquidation threshold and at the target cash level. Again, the target cash level is identified by the super-contact condition, $\lim_{c\uparrow C_V} V''(c) = 0$.

Through equation (13), the equilibrium bid-ask spread that makes the zero-profit condition binding satisfies:

$$\eta(c) = \min\left[\chi, \frac{\gamma}{\delta V(c)} + \kappa\right]. \tag{16}$$

Equation (16) illustrates that larger participation or financing frictions faced by liquidity providers (γ and κ) lead to an increase in the equilibrium bid-ask spread both directly and through their effect on firm value (these effects are quantified in the next subsection). It also captures the real-world observation that the bid-ask spread is wider for less valuable firms, i.e., it is greater for smaller firms as shown, among others, by Hasbrouck (2009) and Chung and Zhang (2014).

Consider now the case in which the bid-ask spread becomes binding at χ. Whenever $\eta = \chi$, investors are indifferent between selling the stock or keeping it. This is consistent with the real-world observation that bid-ask spreads do not increase unboundedly but, especially for small firms, trading volume wanes if bid-ask spreads grow too large. In these cases, firm value satisfies the following ODE:

$$\rho V(c) = (rc + \mu) V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda [V(C_V) - C_V + c - V(c)] - \delta \chi V(c). \qquad (17)$$

Equation (16) suggests that η should become binding at χ if trading firms' participation fees or funding costs are sufficiently large, and firm value is sufficiently low (i.e., it can happen in a right interval of $c = 0$).[19] Whenever $\eta = \chi$ for c close to zero, continuity and smoothness determine how the two ODEs, equations (17) and (15), are pasted together (see Appendix B for further details).

Implications When liquidity provision is endogenous, bid-ask spreads not only affect but also are affected by firm value. In fact, the drop in firm value analyzed in Section 3 (i.e., due to the costs borne by the firm shareholders when trading) is factored into the liquidity providers' zero-profit condition (equation (13)). Specifically, it leads liquidity providers to extract more rents from shocked investors (as a proportion of the value of their claim) to cover participation and funding costs. As a result, the bid-ask spread widens, so trading the stock becomes more expensive for shareholders. The model then shows that frictions borne by liquidity providers are passed on to shareholder and, through this channel, bear an important impact on corporate outcomes.

Insert Figure 1 and 2 Here

[19] As shown in the appendix, there is at most one cash threshold $\underline{C} \in [0, C_V]$ such that: For $c \in [\underline{C}, C_V]$, the proportional bid-ask spread is $\eta(c) \in [0, \chi]$, which solves equation (13); for $c \in [0, \underline{C})$, a fraction δ of shareholders is indifferent between keeping the stock or bearing the bid-ask spread $\eta = \chi$.

Figure 1 and 2 show the endogenous bid-ask spread as well as firm value for different values of the participation fee γ and funding cost κ borne by liquidity providers. The top panels of these figures show that the bid-ask spread is wider when participation fees or funding costs faced by liquidity providers are greater. This result is consistent with the evidence in Anand et al. (2013), who show that liquidity suppliers extract more rents from investors when facing participation or funding costs, a pattern that is particularly strong for small stocks. The result is also in line with Comerton-Forde et al. (2010), who show that variation in the liquidity of a given stock depends, at least in part, on market makers' financial constraints. The top and bottom panels of these figures also illustrate that bid-ask spreads are wider for stocks of less valuable firms, consistent with real-world observations.

Notably, the model shows that frictions affecting the participation of liquidity providers impact the equilibrium bid-ask spread (and, thus, firm value) through a direct and an indirect channel. Consider the effect of an increase in γ or κ. Such an increase implies that liquidity providers will extract a relatively larger share of the value of the claim of shocked shareholders to remain active in the market for the stock. The bid-ask spread borne by liquidity shocked shareholders will then be wider (see equation (13)). This is the direct effect. Yet, this wider bid-ask spread will lead to a reduction in firm value (as shown in Section 3), which then will lead liquidity providers to charge an even larger proportional bid-ask spread. This is the indirect effect.

Quantitatively, an increase in γ from 0.7% to 0.8% would lead to a 9.1% increase in the bid-ask spread if firm value was independent of γ (i.e., if firm value was not affected by such an increase).[20] However, when accounting for its impact on firm value (i.e., the indirect effect illustrated above), the bid-ask spread would be 10.8% bigger as a result of the increase in γ. Similarly, an increase in κ from 0.3% to 0.4% would lead to an increase in the bid-ask spread by 12.2%. When accounting for the effect on firm value, the bid-ask spread increases by 14.4%. The joint impact of such direct and indirect effects provides

[20]For the sake of fixing ideas, firm value is calculated at the midpoint of cash reserves, $c = C_V/2$, in these numerical examples.

an explanation as to why small firms' bid-ask spread increase the most (i.e., much more than those of large firms) when liquidity providers' funding constraints tighten, see for example Anand et al. (2013) and Aragon and Strahan (2012).[21]

Table 4 Here

As larger participation and funding costs lead to larger bid-ask spreads, we would expect them to have an important impact on the firm's cost of internal and external financing and, thus, on their propensity to keep and pay out cash, their risk of forced liquidation, and their willingness to undertake investment opportunities (through the mechanism described in Section 3). Table 4 confirms these results and shows that larger γ or κ exacerbate the firm's financial constraints by increasing the firm's cost of internal and external financing. In addition, the target cash level decreases, cash is paid out more often to shareholders, and the probability of liquidation increases. Table 4 also shows that frictions borne by liquidity providers have a substantial impact on the firm's investment decisions, by leading to a substantial reduction in the maximum price that the firm is willing to pay to increase the cash flow drift from μ to μ_+.[22] Overall, frictions faced by liquidity providers have a substantial, detrimental effect on firm value (as quantified in the last column of Table 4).

Figure 3 Here

Figure 3 further investigates the effects of fγ or κ on the firm probability of liquidation, external financing, and payout. The figure illustrates that the firm's probability of liquidation increases with γ and κ and is greater than in the benchmark case with no bid-ask spreads. In addition, by leading to an increase in the bid-ask spread and, thus, in the firm's cost of external financing, frictions borne by liquidity providers lead to a

[21]Because large firms are traded at small bid-ask spread and have a relatively easy access to several sources of financing, the reinforcing effect between financial constraints and trading frictions should be quantitatively small for these firms.

[22]The analytical expression for the maximum (zero-NPV) cost associated with a given increase in the cash flow drift from μ to μ_+ is provided in Appendix B.

sharp decrease in the firm's probability of external financing, which is lower than in the benchmark case in which the bid-ask spread is zero. Finally, by increasing the opportunity cost of cash, frictions borne by liquidity providers lead to an increase in the firm's payout probability.

All these results illustrate that frictions borne by liquidity providers are eventually passed on to investors and, through the mechanism described in the model, substantially affect corporate policies and outcomes. While this economic mechanism may be quantitatively small for large firms—which have relatively easy access to sources of outside financing, and their stocks are traded at tiny bid-ask spreads and frequently—it is highly relevant for small firms, that indeed face the largest bid-ask spreads in the cross-section and have a more uncertain access to outside financing. This implies that regulation affecting equity markets can affect the value and policies of these firms, as investigated below.

4.2 Applications

4.2.1 Firm-funded Designated Market Makers (DMM)

Theoretical models have shown that competitive market forces may lead to inefficient liquidity provision and suboptimal market outcomes (or market failures), see for instance Bessembinder, Hao, and Zheng (2015).[23] This issue has raised the attention of policy-makers in the past few years. For example, the recommendations of the SEC Advisory Committee on Small and Emerging Companies are based on the idea that competitive market forces may break down when it comes to small or micro caps. Partially addressing this issue, many European markets (like in Germany, France, Italy, the Netherlands, Sweden, and Norway) have contemplated a contract, whereby listed firms pay a DMM to maintain the bid-ask spread below a given (contractual) threshold and enhance the

[23]Specifically, Bessembinder, Hao, and Zheng (2015) show that competitive liquidity provision in secondary markets is associated with reduced welfare and a discounted secondary market price that can potentially dissuade IPOs.

liquidity of the firm's stock. In this section, the model is extended to study the desirability of this policy provision from the perspective of small, financially constrained firms.

Consider the contract between a listed firm and a DMM. Similar to Bessembinder, Hao, and Zheng (2015), I assume that the DMM is required to keep the bid-ask spread within a specific width, in exchange for a rent that is paid by the firm. Consistently, I assume that the firm pays the DMM a periodic payment Γ to keep the bid-ask below a given level $\bar{\eta}$, which is assumed to be smaller than χ to consider the relevant case. That is, $\eta(c) \leq \bar{\eta} < \chi$ for any c. In this setup, the equilibrium bid-ask spread solves:

$$-\delta\,(1-\eta)\;V(c;\eta,C_V(\eta)) + \delta V(c;\eta,C_V(\eta))(1-\kappa) = \gamma - \Gamma \quad s.t. \quad \eta \leq \bar{\eta} < \chi. \tag{18}$$

The above condition differs from (13) in two main aspects. First, the participation fee borne by liquidity providers is partly financed by the firm (i.e, the right-hand side of this equation is just $\gamma - \Gamma$). Second, the contractual Γ is set so to guarantee that the equilibrium η cannot exceed the contractual level $\bar{\eta}$ (see Appendix C). Intuitively, if $\bar{\eta}$ is smaller (meaning that DMM are required to keep the bid-ask spread more narrow), then the periodic fee Γ is larger (i.e., it is more costly to enter the DMM contract from the firm perspective).

By using the results in Section 4.1 together with condition (18), firm value is shown to satisfy the following ODE:

$$(\rho+\delta\kappa)V(c) = (rc+\mu-\Gamma)\,V'(c) + \frac{\sigma^2}{2}V''(c) + \lambda\,[V(C_V) - C_V + c - V(c)] - (\gamma-\Gamma). \tag{19}$$

This equation shows that the periodic flow paid by the firm to liquidity providers reduces the cost of liquidity providers' participation that is passed on to shareholders (the last term on the right-hand side is $\gamma - \Gamma$, whereas it is just γ if the firm does not enter the DMM contract). However, the first term on the right-hand side of equation (19) illustrates that the payment to the DMM, Γ, decreases the firm's periodic cash flow. The following result is shown in Appendix C.

Proposition 5 *For firms facing financing frictions, entering the DMM contract is not value-enhancing, on net.*

Firms face the following tradeoff when deciding whether to enter the DMM contract. On the positive side, entering the DMM contract reduces the bid-ask spread borne by firm shareholders, which in turn decreases the cost of internal and external financing (as illustrated so far in the analysis). On the negative side, funding a DMM drains the firm's cash flows, which in turn makes the firm more financially constrained. Intuitively, because the marginal value of cash is greater than one for financially constrained firms—as illustrated by previous corporate finance model with financing frictions and confirmed in this paper (see Lemma 6 in Appendix A)—cash is more valuable inside the firm than if used to fund the DMM. As a result, the negative effect slightly dominates the positive effect.

Figure 4 Here

Figure 4 compares the equilibrium bid-ask spread and firm value when the firm does and does not enter the DMM contract.[24] The top panel shows that when the firm enters the DMM contract, the equilibrium bid-ask spread decreases, which is beneficial to the firm as it decreases its cost of capital. However, at the same time, the firm has to give up a fraction of its periodic flow of revenues to fund the DMM. On net, the bottom panel of Figure 4 shows that firm value is quite unchanged, on net slightly smaller if the firm finances the DMM, consistent with Proposition (5). In other words, the model predicts that small, financially constrained firms would not derive much benefit from this type of contract.

4.2.2 Financial transaction taxes (FTT)

In the past decade, the European commission has discussed the introduction of a proportional financial transaction tax (FTT) on round-trip transactions (i.e., "Tobin tax").

[24]When the firm does not enter the DMM contract, the bid-ask spread is solved as in Section 4.1.

The main goal of the FTT is to prevent short-term speculation and limit volatility in financial markets. France introduced a 0.2% FTT in 2012, followed by Italy introducing a 0.1% tax in 2013.[25] In March 2019, the U.S. Democrats suggested to impose a 0.1% tax on equity transactions in the attempt to curb high-frequency trading.

Practically, the FTT is a surcharge on trading costs borne by the investors—liquidity providers are exempted from this tax, see Colliard and Hoffmann (2017). While abstracting from general equilibrium aspects, the economic mechanism presented in this paper suggests that this policy proposal would be detrimental to small firms. Intuitively, if investors face increased costs of trading equities because of the FTT, the issuing firms ultimately need to promise a larger return to shareholders. To see this more formally, denote the FTT by ω. Shareholders bear the cost $\eta(c)+\omega$ when selling the stock,[26] where $\eta(c)$ is endogenously set by liquidity providers. As a result, the liquidity providers' zero-profit condition is similar to equation (13) but, differently, the constraint $\eta(c) < \chi - \omega$ needs to hold to make shareholders willing to sell the stock.[27] Whenever $\eta(c) < \chi - \omega$, firm value solves the following ODE (details are collected in Appendix C):

$$(\rho + \delta\kappa + \delta\omega)V(c) = (rc + \mu)\, V'(c) + \frac{\sigma^2}{2}V''(c) + \lambda\left[V(C_V) - C_V + c - V(c)\right] - \gamma. \quad (20)$$

That is, even if the FTT is formally imposed on investors only, it also affects the dynamics of firm value, as it effectively leads to an increase in the return required by the investors (see the left-hand side of equation (20)).

Figure 5 Here

Figure 5 shows the effect of a 0.1% FTT on the equilibrium bid-ask spread as well as on

[25]More precisely, transactions of shares issued by Italian resident companies are to be taxed at a 0.1% rate if executed on-exchange, and at a 0.2% rate if over-the-counter. See Coelho (2016) for additional details.

[26]For tractability, and consistently with our modeling of selling-side frictions only (as discussed in Section 2.1), I assume that the selling party is bearing the tax.

[27]If the opposite inequality held, shocked shareholders would bear the cost of the liquidity shock.

the value of the small firm when investors are subject to the FTT versus the benchmark case in which they are not. Because the FTT is ultimately borne by shareholders, it increases the return that the firm needs to promise to investors, which in turn reduces firm value. As illustrated in Section 3, the firm will be more subject to forced liquidation, will keep less cash, will forgo profitable investment opportunities. Through the mechanism illustrated in Section 4.1, this drop in firm value enters the liquidity providers' zero-profit condition. For a given participation or funding cost, the reduction in firm value implies that competitive trading firms need to extract a larger fraction of firm value to cover the participation cost. That is, even if the FTT does not directly affect trading firms, it prompts them to charge a larger cost of liquidity provision. In fact, the top panel shows that the endogenous bid-ask spread set by competitive liquidity providers is greater in the presence of the FTT.

Overall, this analysis suggests that the FTT would have detrimental real effect on small, financially constrained firms. That said, the analysis is silent on the desirability (the welfare gains) arising from this tax, in that the focus of the paper is on the effects on the corporate sector (and importantly, it abstracts from speculative trading and other forces that could make the tax desirable). The model suggests that if a FTT is beneficial to warrant the stability of financial markets, then a provision making the magnitude of the FTT contingent on a firm's market capitalization could help relieve the potentiallly harmful real effects illustrated above.

5 Robustness

5.1 Providing liquidity via share repurchases

The analysis shows that trading costs translate into a larger cost of capital, which exacerbates the firm's financial constraints. A question arises as to whether it would be optimal for the firm to commit to repurchasing shares of shocked shareholders, thus acting as a liquidity provider and decrease the liquidity premium required by shareholders. To clearly single out the firm's tradeoffs, we consider the case in which the bid-ask spread is exogenous.

Suppose that the firm follows this policy and repurchases the shares of shocked investors at their fair value, meaning that the firm would have a constant cash outflow equal to $\delta V(c)$ on any time interval. Firm value would satisfy

$$\rho V(c) = [rc + \mu - \delta V(c)]\, V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda \left[V(C_V) - C_V + c - V(c) \right] , \qquad (21)$$

subject to the same boundary conditions in Section 3.1. Equation (21) differs from (5) in that there is no loss borne by shocked investors because of the firm's commitment to buy back shares. Yet, such a commitment would decrease firm value. In fact, the flow cost of repurchasing shares of shocked shareholders (see the first term on the right-hand side of this equation, which differs from equation (6) as it includes the term $\delta V(c)V'(c)$) is larger than the cost of investors' trading frictions on the firm's cost of capital, $\delta V(c)\eta$, because the marginal value of cash is greater inside the firm than if paid out, for any $c < C_V$ (and so $V'(c) \geq 1 > \eta > 0$; see Lemma 6 in Appendix A).[28]

Alternatively, management could buy back the shares of shocked shareholders at a price smaller than $\delta V(c)$. Yet, this price cannot be smaller than $\delta V(c)(1-\eta)$, otherwise shocked shareholders would sell the stock to the intermediaries. That is, management would have to buy back shares at a price at least equal to $\delta V(c)(1-\eta)$. When following this policy, firm value would satisfy

$$(\rho + \delta\eta) V(c) = [rc + \mu - (1-\eta)\delta V(c)]\, V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda \left[V(C_V) - C_V + c - V(c) \right]$$

where the last term denotes the loss borne by shocked shareholders. A direct comparison of this equation with (6) highlights that this policy decreases firm value too. In fact, the liquidity premium paid by the firm, together with the flow cost of repurchasing shares,

[28] This tradeoff of costs and benefits is similar to that faced by the firm when deciding whether to enter the DMM contract (see Section 4.2.1).

decreases firm value more than the liquidity premium paid when following the policy in Section 3.1 (i.e., not committing to repurchase shares for $c < C_V$). In fact, not only the liquidity premium would be greater, but the commitment to repurchasing shares would drain the firm's cash flows. It is then easy to generalize that repurchasing shares at prices in the interval $[\delta T(c), \delta V(c)]$ is suboptimal whenever $c < C_V$.

These results imply that payouts (share repurchases or dividends) are suboptimal when cash reserves are below the target level. The reason is that cash is more valuable inside the firm ($V'(c) > 1$) than outside for any $c < C_V$. Instead of repurchasing shares for any c, the optimal policy implies that the firm "provides liquidity" to shareholders by reducing the target cash level with respect to the benchmark case with no trading costs (as from Proposition 1), meaning that the payout threshold can be hit more often.

5.2 Debt financing (bank credit lines)

Firms can also access liquidity by drawing funds from bank credit lines (see Sufi, 2009). In particular, small firms typically do not have access to the corporate bond and commercial paper markets, and are more likely to tap debt financing by drawing funds from bank lines of credit. A credit line is a source of funding that the firm can access at any time up to a pre-established limit, which I denote by L. Whenever the credit limit is finite ($L < \infty$), the firm has a positive demand for cash. As shown by BCW, this is true for exogenous or endogenous (value-maximizing) L.[29] In this section, I assess the model results in the presence of this additional source of financing.

I follow BCW by assuming that the firm pays a constant spread, β, over the risk-free rate on the amount of credit used. Because of this cost, it is optimal for the firm to tap the credit line only when cash reserves are exhausted. The firm then uses cash as the marginal source of financing if $c \in [0, C_V(L)]$ (the cash region), where $C_V(L)$ denotes the target cash level in this environment. Conversely, the firm draws funds from the credit

[29] Firms often face credit supply frictions that prevent them from taking the value-maximizing limit L. Endogenizing L is an interesting extension to understand the relation between stock liquidity and the firm's willingness to access bank credit, and I leave it for future research.

line when $c \in [-L, 0]$ (the credit line region). Firm value satisfies (6) in the cash region, whereas it satisfies

$$(\rho + \delta\eta) V(c) = [(\rho + \beta)c + \mu] V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda [V(C_V) - V(c) - C_V + c] \qquad (22)$$

in the credit-line region. On top of the smooth-pasting and super-contact conditions at $C_V(L)$ similar to (8) and (9), the system of ODEs (6)–(22) is solved subject to the following boundary conditions. The first condition, $V(-L) = \max[\ell - L, 0]$, means that if $\ell \geq L$, the credit line is fully secured and shareholders are residual claimants in liquidation. Moreover, the conditions $\lim_{c\uparrow 0} V(0) = \lim_{c\downarrow 0} V(0)$ and $\lim_{c\uparrow 0} V'(0) = \lim_{c\downarrow 0} V'(0)$ guarantee continuity and smoothness at the point where the cash and the credit line regions are pasted.

Insert Figure 6 Here

Figure 6 compares the impact of bid-ask spreads when a firm does and does not have access to bank credit. I use the same parametrization in Table 1 and additionally set $L = 0.08$ and $\beta = 1.5\%$ (in line with Sufi, 2009). The figure shows that the effects of bid-ask spreads on corporate policies are similar irrespective of the firm's access to bank credit. Access to credit relaxes the precautionary need to keep cash and leads to a decrease in the target cash level. However, the presence of the bid-ask spread reduces this target level below the benchmark with no trading costs (in which the target cash level is driven by precautionary motivations only). The probability of liquidation and investment decisions are almost the same irrespective of the firm's access to credit lines.

6 Concluding remarks

Two key features characterize small firms: They are gripped by financing frictions, and trading their stocks entails non-negligible bid-ask spreads. This paper develops a model that jointly analyzes these features. The model shows that bid-ask spreads increase the firms' cost of external *and* internal financing, making firms more financially constrained (as they keep less cash reserves and are less likely to raise equity) and more exposed to forced liquidations. These firms are also more likely to forgo investment opportunities, as the additional return required by the investors erodes the profitability of investment opportunities. Overall, these effects decrease firm value.

The model also shows that when liquidity providers face participation frictions and set the bid-ask spread competitively, this drop in firm value feeds back into the magnitude of the bid-ask spread. This mechanism implies that frictions faced by liquidity providers are passed on to investors and, through this channel, have an important impact on the policies, financial constraints, and survival rates of small firms—then providing a rationale for why small firms are fading from exchanges. Overall, the model suggests that the architecture of secondary market transactions has a prime effect on corporate decisions, especially for firms that are faced with large bid-ask spread and severe financing frictions, like small and micro firms. The model then provides a framework that can help analyze the effects of some recent regulation targeting financial markets from the corporate perspective, like designated market makers or financial transaction taxes.

Appendices

A Proof of the results in Section 3

Throughout the Appendix, I define the quantity

$$\Phi \equiv \delta\eta$$

to ease the notation.

I start by proving that $V(c)$ is increasing and concave for any $c < C_V$.

Lemma 6 *$V'(c) > 1$ and $V''(c) < 0$ for any $c \in [0, C_V)$.*

Proof. Simply differentiating the following equation (which is equivalent to equation (6))

$$(\rho + \lambda + \Phi)\ V(c) = V'(c)\,(rc + \mu) + \frac{\sigma^2}{2} V''(c) + \lambda \Big[V(C_V) - C_V + c \Big], \tag{23}$$

one gets

$$(\rho + \lambda + \Phi - r)\, V'(c) = V''(c)\,(rc + \mu) + \frac{\sigma^2}{2} V''(c) + \lambda.$$

By the conditions $V'(C_V) = 1$ and $V''(C_V) = 0$, it follows that $V'''(C_V) = \frac{2}{\sigma^2}(\rho + \Phi - r) > 0$ as $r < \rho$, meaning that there exists a left neighborhood of C_V such that for any $c \in (C_V - \epsilon, C_V)$, with $\epsilon > 0$, the inequalities $V'(c) > 1$ and $V''(c) < 0$ hold. Toward a contradiction, I assume that $V'(c) < 1$ for some $c \in [0, C_V - \epsilon]$. Then there exists a point $C_c \in [0, C_V - \epsilon]$ such that $V'(C_c) = 1$ and $V'(c) > 1$ over (C_c, C_V), so

$$V(C_V) - V(c) > C_V - c \tag{24}$$

for any $c \in (C_c, C_V)$. For any $c \in (C_c, C_V)$ it must be also that

$$V''(c) = \frac{2}{\sigma^2} \left\{ (\rho + \lambda + \Phi)\, V(c) - [rc + \mu] V'(c) - \lambda (V(C_V) + c - C_V) \right\}$$

Using (24), jointly with $V(C_V) = \frac{rC_V + \mu}{\rho + \Phi}$, it follows that

$$V''(c) < \frac{2}{\sigma^2} \left\{ (\rho + \Phi)(V(C_V) + c - C_V) - rc - \mu \right\} = \frac{2}{\sigma^2}(c - C_V)(\rho + \Phi - r) < 0.$$

This means that $V'(c)$ is decreasing for any $c \in (C_c, C_V)$, which contradicts $V'(C_c) = V'(C_V) = 1$. It follows that C_c cannot exists. So, $V'(c) > 1$ and $V''(c) < 0$ for any $c \in [0, C_V)$, and the claim follows. ■

A.1 Proof of Proposition 1

In this section, I express the function $V(c)$ as a function of X, denoting the threshold satisfying $V'(X,X) - 1 = V''(X,X) = 0$. To prove the claim, I exploit the following auxiliary results.

Lemma 7 *The function $V(c,X)$ is decreasing in the payout threshold X.*

Proof. To prove the claim, I take $X_1 < X_2$, and I define the auxiliary function $k(c) = V(c,X_1) - V(c,X_2)$, that satisfies

$$(\rho + \Phi + \lambda)k(c) = (rc+\mu)k'(c) + 0.5\sigma^2 k''(c) + \lambda(X_1 - X_2)[r/(\rho+\Phi) - 1] \tag{25}$$

for any $c \in [0, X_1]$. By previous result and straightforward calculations, the function is positive at X_2 as $k(X_2) = (X_1 - X_2)[r/(\rho+\Phi)-1] > 0$. By the definition of X_1 and X_2, the function $k(c)$ is decreasing and convex for $c \in [X_1, X_2)$. Therefore, $k(X_1) > 0$. Note that the function cannot have a negative local minimum on $[0, X_1]$ because the last term on the right hand side of (25) is positive. In addition, the function $k'(c)$ does not have neither a positive local maximum nor a negative local minimum, otherwise the equation $(\rho+\Phi+\lambda-r)k'(c) = (rc+\mu)k''(c) + 0.5\sigma^2 k'''(c)$ would not hold (respectively $k'(c) > 0 = k''(c) > k'''(c)$ and $k'(c) < 0 = k''(c) < k'''(c)$ at a positive maximum and at a negative minimum). As k is convex at X_1, this means that k' is increasing at X_1, and therefore it must be negative for any $c \in [0, X_1]$. Jointly with $k(X_1) > 0$, this means that $k(c) > 0$ for any $c \in [0, X_2]$. The claim follows. ■

Lemma 8 *For a given payout threshold X and two given $\eta_1 > \eta_2$, $V(c, X, \eta_2) > V(c, X; \eta_1)$ for any $c \in [0, X]$.*

Proof. As in the previous section, I define $\Phi_i = \delta\eta_i$ with $i = 1, 2$, and the auxiliary function $h(c) = V(c, X; \eta_2) - V(c, X; \eta_1)$. I need to prove that, for a given payout threshold X, $h(c) > 0$ for any $c \in [0, X]$. At X, the function is positive as

$$h(X) = (rX+\mu)\left(\frac{1}{\rho+\Phi_2} - \frac{1}{\rho+\Phi_1}\right) = (rX+\mu)\frac{\Phi_1 - \Phi_2}{(\rho+\Phi_1)(\rho+\Phi_2)} > 0,$$

because $\Phi_1 > \Phi_2$ as $\eta_1 > \eta_2$, and $h'(X) = h''(X) = 0$. In addition, the function satisfies

$$[rc+\mu]h'(c) + \frac{\sigma^2}{2}h''(c) - (\rho+\lambda+\Phi_2)h(c) + \lambda h(X) = (\Phi_2 - \Phi_1)V(c, X; \chi_1)$$

and the right hand side is negative. Differentiating gives $[rc+\mu]h''(c) + \frac{\sigma^2}{2}h'''(c) - (\rho+\lambda+\Phi_2 - r)h'(c) = (\Phi_2 - \Phi_1)V_s'(c, X; \chi_1)$. At X, I get $\frac{\sigma^2}{2}h'''(X) = \Phi_2 - \Phi_1$, meaning that $h'''(X) < 0$. This means that the second derivative is decreasing in a neighbourhood of X, so one has $h''(c) > 0$ in a left neighbourhood of X. In turn, this means that $h'(c)$ is increasing in such a neighbourhood of X, then implying that $h'(c) < 0$ in a left neighbourhood of X. Now I need to prove that the function is decreasing for any c smaller that X. Note that, by the ODE above, $h'(c)$ cannot have a negative local minimum. As $h'(X) = 0$ and it is negative and increasing in a left neighbourhood of X, this means that $h'(c)$ should be negative for any $c < X$, so $h(c)$ is always decreasing. As it is positive at X, it means that it should be always positive, so $h(c) > h(X) > 0$ so it is positive for any $c < X$. ■

Exploiting the results above, I can prove the following lemma.

Lemma 9 *For any* $\eta_1 > \eta_2$, $C_V(\eta_1) < C_V(\eta_2)$.

Proof. The payout thresholds $C_V(\eta_1)$ and $C_V(\eta_2)$ are the unique solution to the boundary conditions $V(0, C_V(\eta_2); \eta_2) - \ell = 0 = V(0, C_V(\eta_1); \eta_1) - \ell$. Exploiting the result in Lemma 8, I now take, for instance, $X = C_V(\eta_1)$. It then follows that

$$V(0, C_V(\eta_1); \eta_2) - \ell \; > \; 0 \; = \; V(0, C_V(\eta_1); \eta_1) - \ell.$$

As V is decreasing in the payout threshold, this means that $C_V(\eta_1) < C_V(\eta_2)$ to get the equality $\ell - V(0, C_V(\eta_2); \eta_2) = 0$. The claim follows. ■

The next results stem from Lemma 9.

Corollary 10 *When the bid-ask spread is positive, the target cash level is lower than in the benchmark case with no bid-ask spread, i.e.* $C_V < C^*$.

Note also that all the results in this section can be extended for two parameters $\delta_1 > \delta_2$. The following result is then straightforward.

Corollary 11 *For any* $\delta_1 > \delta_2$, $C_V(\delta_1) < C_V(\delta_2)$.

A.2 Proof of Proposition 2

Using the insights from Dixit and Pyndick (1994), the dynamics of $P_p(c, X)$ are given by

$$\begin{aligned} & P_p'(c)(rc + \mu) + \frac{\sigma^2}{2} P_p''(c) - \lambda \, P_p(c) = 0 \\ \text{s.t. } & P_p(0) = 0 \\ & P_p(X) = \; 1. \end{aligned}$$

The first boundary condition implies that when the controlled cash process is absorbed at zero, the firm liquidates and the payout probability is zero. The second boundary condition is obvious given that cash is paid out at X. The following lemma shows that greater bid-ask spreads are associated with larger payout probability.

Lemma 12 *For any* $\eta_1 > \eta_2$, $P_p(c, C_V(\eta_1)) \geq P_p(c, C_V(\eta_2))$.

Proof. By Lemma 9, $C_V(\eta_1) < C_V(\eta_2)$. To ease the notation throughout the proof, I define $X_1 \equiv C_V(\eta_1)$ and $X_2 \equiv C_V(\eta_2)$. Consider the function

$$h(c) = \; P_p(c, X_1) - P_p(c, X_2).$$

Because of the boundary conditions at zero and X_1, $h(0) = \; 0$ and $h(X_1) = 1 - P_p(c, X_2) > 0$. This means that the function is null at the origin, and positive at C_V. Note that $h(c)$ cannot have neither a positive local maximum ($h(c) > 0$, $h'(c) = 0$, $h''(c) < 0$) nor a negative local minimum ($h(c) < 0$, $h'(c) = 0$, $h''(c) > 0$) on $[0, X_1]$, as otherwise the equation $h''(c)\frac{\sigma^2}{2} + h'(c)[rc + \mu] - \lambda h(c) = 0$ would not hold. Therefore, the function must be always positive and increasing over the relevant interval, and the claim follows. ■

The result below is a straightforward consequence of Lemma 12 and the fact that, in the absence of trading costs, $\eta = 0$ (or $\delta = 0$).

Corollary 13 *When trading the firm's stock is costly, the payout probability P_p is larger than in the benchmark case with no trading costs, i.e. $P_p(c, C^*) < P_p(c, C_V)$.*

These results can be extended for two parameters $\delta_1 > \delta_2$, as follows.

Corollary 14 *For any $\delta_1 > \delta_2$, $P_p(c, C_V(\delta_1)) \geq P_p(c, C_V(\delta_2))$.*

A.3 Proof of Proposition 3

I derive the results regarding the probability of liquidation $P_l(c, X)$, because the probability of external financing is just $P_f(c, X) = 1 - P_l(c, X)$. Using standard methods (see e.g., Dixit and Pyndick, 1994), the dynamics of $P_l(c, X)$ are given by

$$P_l'(c)(rc + \mu) + \frac{\sigma^2}{2} P_l''(c) - \lambda\, P_l(c) = 0$$

$$\text{s.t. } P_l(0) = 1 \tag{26}$$

$$P_l'(X) = 0, \tag{27}$$

where the first boundary condition is given by the definition of P_l, while the second boundary condition is due to reflection at the payout threshold.

Now I prove that the probability of liquidation is higher when the firm's stocks are illiquid. To do so, I first prove that the probabilities $P_l(c, C^*)$ and $P_l(c, C_V)$ are decreasing and convex in c. In the following, I employ the generic function $P_l(c, X) \equiv P_l(c)$.

Lemma 15 *The probability $P_l(c, X)$ is decreasing and convex for any $c \in [0, X]$.*

Proof. To prove the claim, I exploit arguments analogous to those of Lemma 6. As $P_l'(X) = 0$ and $P_l(X) \geq 0$, it must be that $P_l''(X) > 0$. Then, there exists a left neighbourhood of X, $[X - \epsilon, X]$ with $\epsilon > 0$, over which $P_l'(c) < 0$ and $P_l''(c) > 0$. Toward a contradiction, suppose that there exists some $c \in [0, X - \epsilon]$ where $P_l'(c) > 0$. Then, there should be a $\bar{C}$ such that $P_l'(\bar{C}) = 0$, while $P_l'(c) < 0$ for $c \in [\bar{C}, X]$. For any $c \in [\bar{C}, X]$ it must be that

$$P_l''(c) = \frac{2}{\sigma^2}\left[\lambda P_l(c) - P_l'(c)(rc + \mu)\right] > \frac{2}{\sigma^2}\,\lambda P_l(X) \; > \; 0\,.$$

Then, $P_l''(c) > 0$ for any $c \in [\bar{C}, X]$ means that $P_l'(c)$ is always increasing on $c \in [\bar{C}, X]$, contradicting $P_l'(\bar{C}) = P_l'(X) = 0$. The claim follows. ■

Now I prove that $P_l(c, C_V) \geq P_l(c, C^*)$.

Lemma 16 *For any $\eta_1 > \eta_2$, $P_l(c, C_V(\eta_1)) \geq P_l(c, C_V(\eta_2))$.*

Proof. By Lemma 9, $C_V(\eta_1) < C_V(\eta_2)$. To ease the notation throughout the proof, I define $X_1 \equiv C_V(\eta_1)$ and $X_2 \equiv C_V(\eta_2)$. By Lemma 15, the functions $P_l(c, X_1)$ and $P_l(c, X_2)$ are positive, decreasing and convex over the interval of definition. I define the auxiliary function

$$h(c) = P_l(c, X_1) - P_l(c, X_2).$$

Note that $h(c)$ cannot have neither a positive local maximum ($h(c) > 0$, $h'(c) = 0$, $h''(c) < 0$) nor a negative local minimum ($h(c) < 0$, $h'(c) = 0$, $h''(c) > 0$) on $[0, X_1]$, as otherwise the equation $h''(c)\frac{\sigma^2}{2} + h'(c)[rc + \mu] - \lambda h(c) = 0$ would not hold. In addition, $h(0) = 0$, and $h'(X_1) = -P_l'(c, X_2) > 0$ because of the boundary conditions at zero and at X_1. This means that the function is null at the origin, and increasing at C_V. Toward a contradiction, assume that $h(X_1)$ is negative. This would imply the existence of a negative local minimum, given that the function is null at zero and it is increasing at X_1. This cannot be the case as argued above, contradicting that $h(X_1) < 0$. Therefore, the function must be always positive, and the claim follows. ■

The result below is a straightforward consequence of Lemma 16 and the fact that, in the absence of trading costs, $\eta = 0$ (or $\delta = 0$).

Corollary 17 *When trading the firm's stock is costly, the probability of liquidation P_l is larger than in the benchmark case with no trading costs, i.e. $P_l(c, C^*) < P_l(c, C_V)$.*

These results can be extended for two parameters $\delta_1 > \delta_2$, as follows.

Corollary 18 *For any $\delta_1 > \delta_2$, $P_l(c, C_V(\delta_1)) \geq P_l(c, C_V(\delta_2))$.*

A.4 Proof of Proposition 4

I exploit the dynamic programming result in Décamps and Villeneuve (2007) and HMM, establishing that the growth option has a non-positive NPV if and only if $V(c) > V_+(c - I)$ for any $c \geq 0$, where I denote by $V_+(c - I)$ the value of the firm after investment. To prove the claim, I rely on the following lemma.

Lemma 19 *$V(c) \geq V_+(c - I)$ for any $c \geq I$ if and only if $I \geq I_V$, where I_V is defined as in Proposition 4.*

Proof. I define $\bar{c} = \max[C_V, I + C_{V+}]$. The inequality $V(c) \geq V_+(c - I)$ for $c > \bar{c}$ means that $c - C_V + V(C_V) \geq c - C_{V+} - I + V_+(C_{V+})$. Using the definition of I_V, the former inequality is equivalent to the inequality $I \geq I_V$, by straightforward calculations.

To prove the sufficient condition, I can just prove that $V(c) \geq V_+(c - I_V)$ for any $c \geq I_V$. I exploit the inequalities $C_V < C_{V+} + I_V$ and $\mu_+ - \mu - rI_V > 0$ (these inequalities stem from a slight modification of Lemma C.3 in HMM, so I omit the details). For $c \geq C_V$, the following inequality

$$V_+(c - I_V) \leq V_+(C_{V+}) + c - I_V - C_{V+} = c - C_V + V(C_V) = V(c)$$

holds. The first inequality is due to the concavity of V_+, the first equality is given by the definition of I_V, whereas the second equality is due to the linearity of V above C_V. I now need to prove the result for $c \in [I_V, C_V]$. To this end, I define the auxiliary function $u(c) =$

$V(c) - V_+(c - I_V)$. The function $u(c)$ is positive at C_V as argued above, $u'(C_V) < 0$ and $u''(C_V) > 0$. On the interval of interest it satisfies

$$(\rho + \Phi + \lambda)u(c) = (rc + \mu)u'(c) + \frac{\sigma^2}{2}u''(c) + (\mu + rI_V - \mu_+)V'_+(c - I_V) + \lambda(V(C_V) - C_V - V_+(C_{V+}) + C_{V+} + I_V)$$

where the last term on the right hand side is zero by the definition of I_V, while the third term is negative. Then, the function cannot have a positive local maximum here, because otherwise $u(c) > 0$, $u''(c) < 0 = u'(c)$, and the ODE above would not hold. Jointly with the fact that $u(C_V)$ is positive, decreasing and convex means that the function is always decreasing on this interval. Then, $u(c)$ is also always positive, and the claim holds. ■

B Proof of the results in Section 4.1

Two separate cases can be considered depending on the relative magnitude of participation or funding costs borne by liquidity providers. In the following, these two cases are considered separately.

Case $\eta(c) < \chi$ for any c. This is the case if participation and funding costs are sufficiently low, so that condition

$$\frac{\gamma}{\delta(\chi - \kappa)} \leq \ell \tag{28}$$

holds. When this is the case, firm value satisfies equation (15) for any $c < C_V$, and firm value is solved subject to the boundary condition at the liquidation threshold and at C_V as reported in the main text. As a straightforward extension of Lemma 6, it is possible to show that $V(c)$ is increasing and concave in cash reserves.

Case $\eta(c) = \chi$ for some c. This is the case if participation and funding costs are sufficiently large, so that condition (28) does not hold. Because firm value is increasing in c, there is at most one cash threshold $\underline{C} \in [0, C_V]$ such that the proportional bid-ask spread is $\eta(c) \in [0, \chi]$ for $c \in [\underline{C}, C_V]$ and solves equation (13). For $c \in [0, \underline{C})$, a fraction δ of shareholders is indifferent between keeping the stock of selling it at price $\chi V(c)$. Continuity and smoothness at $\underline{C}$ mean that the system of equations (15) and (17) is solved subject to the following conditions:

$$\lim_{c \uparrow \underline{C}} V(c) = \lim_{c \downarrow \underline{C}} V(c) \quad \text{and} \quad \lim_{c \uparrow \underline{C}} V'(c) = \lim_{c \downarrow \underline{C}} V'(c)$$

on top of the boundary conditions at the liquidation and payout threshold ($V(0) - \ell = \lim_{c \uparrow C_V} V'(c) - 1 = \lim_{c \uparrow C_V} V''(c) = 0$). In this case too, I show next that the value function is strictly monotone and concave over $0 \leq c < C_V$. Differentiating equation (15) gives the following ODE

$$(\rho + \lambda + \delta\kappa - r)\, V'(c) = V''(c)\,(rc + \mu) + \frac{\sigma^2}{2}V'''(c) + \lambda.$$

Jointly with the boundaries $V'(C_V) = 1$ and $V''(C_V) = 0$, this ODE implies that $V'''(C_V) > 0$, meaning that there exists a left neighborhood of C_V such that for any $c \in (C_V - \epsilon, C_V)$, with

$\epsilon > 0$, the inequalities $V'(c) > 1$ and $V''(c) < 0$ hold. Toward a contradiction, I assume that $V'(c) < 1$ for some $c \in [0, C_V - \epsilon]$. Then, there should be a point $C_c \in [0, C_V - \epsilon]$ such that $V'(C_c) = 1$ and $V'(c) > 1$ over (C_c, C_V), so $V(C_V) - V(c) > C_V - c$ for any $c \in (C_c, C_V)$. The point C_c could belong either to the interval $[0, \underline{C}]$ or in the interval $[\underline{C}, C_V]$. I now discriminate between these two cases. If $\underline{C} < C_c < C_V$, it must be that for any $c \in (C_c, C_V)$

$$V''(c) = \frac{2}{\sigma^2}\left\{(\rho + \lambda + \delta\kappa)\, V(c) - (rc + \mu)V'(c) + \gamma - \lambda[V(C_V) + c - C_V]\right\}$$

Using $V(C_V) - V(c) > C_V - c$, jointly with $V(C_V) = \frac{rC_V + \mu - \gamma}{\rho + \delta\kappa}$, it follows that

$$V''(c) < \frac{2}{\sigma^2}\left\{(\rho + \delta\kappa)(V(C_V) + c - C_V) - rc - \mu + \gamma\right\} = \frac{2}{\sigma^2}(c - C_V)(\rho + \delta\kappa - r) < 0.$$

This means that $V'(c)$ is decreasing for any $c \in (C_c, C_V)$, which contradicts $V'(C_c) = V'(C_V) = 1$. So, $V'(c) > 1$ for any $c \in [\underline{C}, C_V)$, so such C_c does not exist on $[\underline{C}, C_V]$.

I now consider the case $0 < C_c < \underline{C}$. Should such point C_c exist, the strict concavity of $V(c)$ over $[\underline{C}, C_V]$ means that there should be a maximum $C_m \in [C_c, \underline{C}]$ for the first derivative over the interval $(C_c, \underline{C})$, such that $V'(C_m) > 1$, $V''(C_m) = 0$ and $V'''(C_m) < 0$. Differentiating equation (17) gives

$$V''(c)[rc + \mu] + V'''(c)\frac{\sigma^2}{2} - V'(c)(\rho + \delta\chi - r) + \lambda(1 - V'(c)) = 0.$$

Then, $V'''(C_m)\frac{\sigma^2}{2} = (\rho + \delta\chi - r)V'(C_m) + \lambda(V'(C_m) - 1) > 0$, which contradicts the existence of such a maximum C_m for $V'(c)$. It follows that C_c cannot exists, and the claim follows.

Investment decision with endogenous liquidity provision Following arguments similar to those in Appendix A.4, the firm finds it optimal to invest in the growth option if it has positive NPV. This is the case if $V(c) > V_+(c - I)$, where again I denote by $V_+(c - I)$ the value of the firm after investment. Thus, a straightforward modification of Lemma 19 implies that the zero-NPV investment (i.e., that makes the NPV of the project equal to zero) is given by the following expression:

$$I_V = \frac{\mu_+ - \mu}{\rho + \delta\kappa} - \left(1 - \frac{1}{\rho + \delta\kappa}\right)(C_{V+} - C_V) \tag{29}$$

where I denote by C_{V+} the target cash level after investment (i.e., when the cash flow drift is μ_+).

B.1 Endogenizing the mass of active intermediaries

In this Section, I assess the robustness of the results in Section 4 to an alternative model specification. I keep the assumptions that intermediaries' constant market presence entails a flow cost γ and that the market for liquidity provision is perfectly competitive. Differently, I assume that the measure of active intermediaries is endogenous.

Intermediaries are an infinite and atomless mass. Because of participation costs, however, only a finite measure, θ_t, is active. The measure of intermediaries in the market of the stock determines the probability with which shocked shareholders trade with these liquidity providers rather than keeping the asset). I define this probability as

$$\pi_t \equiv \frac{\theta_t}{\alpha + \theta_t}, \tag{30}$$

where $\alpha > 0$ captures inefficiencies that reduce this probability (e.g., rigidities in trading protocols or technological deficiencies). This specification of π_t implies that shocked shareholders never opt for their outside option if θ_t tends to infinity, as in the environment analyzed in Section 3. This specification captures the notion of competition by order flow, as the probability with which intermediaries contact investors, $\frac{\pi_t}{\theta_t}$, decreases with θ_t.

Competition in the market for liquidity provision means that the measure of active intermediaries in the market of the stock is determined by the zero-profit condition:

$$\frac{\pi(\theta)}{\theta}\delta\left[V(c)(1-\kappa) - (1-\eta)V(c)\right] = \gamma \tag{31}$$

which admits an interpretation analogous to (13). Simplifying gives

$$\frac{\delta(\eta-\kappa)}{\alpha+\theta_t}V(c) = \gamma. \tag{32}$$

So,

$$\theta(c) = \left(\frac{\delta(\eta-\kappa)}{\gamma}V(c) - \alpha\right)^+ \pi(c) = \left(1 - \frac{\alpha\gamma}{\delta(\eta-\kappa)V(c)}\right)^+. \tag{33}$$

The measure of active intermediaries and the ensuing probability must be non-negative, which implies that intermediaries participate in the market for the stock if the value of equity is larger than a critical value defined by

$$\underline{V} = \frac{\alpha\gamma}{\delta(\eta-\kappa)}.$$

This critical value increases with the frictions faced by intermediaries, γ and κ.

The active measure of liquidity providers affects the cost of shareholders' trading. This cost, in turn, affects firm value. Similar to Section 3, I conjecture the existence of a target cash level C_V in this environment. Following arguments similar to those exploited in the proof of Section 4, it is possible to show that there is at most one threshold $\underline{C} \in [0, C_V]$ such that $V(\underline{C}) = \underline{V}$. For any $c > \underline{C}$, the measure of active intermediaries $\theta(c)$ (and the probability $\pi(c)$) is positive, non-decreasing, and concave in c.

For any $c < \underline{C}$, shocked shareholders bear the cost $\delta\chi V(c)$ on aggregate (as they are not able to sell the stock to trading firms). Firm value satisfies the following ODE:

$$(\rho + \delta\chi)\,V(c) = (rc+\mu)\,V'(c) + \frac{\sigma^2}{2}V''(c) + \lambda\left[V(C_V) - C_V + c - V(c)\right]. \tag{34}$$

This equation admits an interpretation analogous to (6), but the return required by the investors (the left-hand side) is larger and equal to $\rho+\delta\chi$. The reason is that declining liquidity provision in the market of the stock increases trading costs to shocked shareholders, and so increases the illiquidity premium and the firm's cost of capital.

For any $c > \underline{C}$, the expected loss associated with a liquidity shock is

$$\big[\pi(c)\eta\delta V(c) + \big(1-\pi(c)\big)\chi\delta V(c)\big]\,dt$$

The first term represents the probability-weighted loss when selling the stock to an active trading firm, whereas the second term represents the probability-weighted loss associated with keeping the stock. For any $c \geq \underline{C}$, firm value satisfies

$$(\rho + \delta\eta)\,V(c) = (rc+\mu)\,V'(c) + \frac{\sigma^2}{2}V''(c) + \lambda\left[V(C_V) - C_V + c - V(c)\right] - \frac{\alpha\gamma(\chi-\eta)}{(\eta-\kappa)}. \quad (35)$$

Equation (35) shows that the return required by the investors (the left-hand side) is the same as equation (6). Yet, the last term on the right-hand side reveals that intermediaries' participation frictions matter to shareholders. These frictions hamper liquidity provision in the market of the stock and so increase the expected trading costs incurred by shocked shareholders. This term is akin to a flow cost, which is larger if intermediaries' participation frictions, α and γ, increase.

Continuity and smoothness at $\underline{C}$ mean that the system of equations (34)–(35) is solved subject to the following conditions:

$$\lim_{c\uparrow\underline{C}} V(c) = \lim_{c\downarrow\underline{C}} V(c) \quad \text{and} \quad \lim_{c\uparrow\underline{C}} V'(c) = \lim_{c\downarrow\underline{C}} V'(c)\,.$$

In addition, $V(c)$ satisfies

$$V(0) - \ell = \lim_{c\uparrow C_V} V'(c) - 1 = 0$$

at the liquidation threshold and at the target cash level, similar to Section 3.1. Lastly, the target cash level is identified by the super-contact condition, $\lim_{c\uparrow C_V} V''(c) = 0$.

C Proof of the result in Section 4.2

C.1 Proof of the analysis in Section 4.2.1 (DMM)

Consider the zero-profit condition if the firm enters the DMM contract. The equilibrium payment Γ needs to guarantee that the maximum contractual bid-ask spread is $\bar{\eta} < \chi$. As shown in the analysis in Section 4.1, the bid-ask spread decreases as firm value increases. Thus, the minimum Γ that the firm needs to pay to the DMM solves the following equation holds:

$$\delta V(0)(1-\kappa) - \delta(1-\bar{\eta})V(0) + \Gamma = \gamma, \quad (36)$$

which gives

$$\Gamma^* = \gamma - \delta(\bar{\eta} - \kappa)V(0).$$

In this environment, the bid-ask spread is given by the following expression:

$$\eta(c) = \frac{\gamma - \Gamma^*}{\delta V(c)} + \kappa. \quad (37)$$

which indeed is lower than the equilibrium bid-ask when the firm does not enter the DMM contract. Next, I prove Proposition 5, showing that, on net, entering the DMM contract cannot increase firm value.

Proof. Comparing equation (19) with equation (15) illustrates that the two equations only differ for the additional term $-\Gamma^* V'(c) + \Gamma^*$ on the right-hand side. This expression can be rewritten as $\Gamma^*(1 - V'(c))$. As $V'(c) \geq 1$ for any $c < C_V$, the expression $\Gamma^*(1 - V'(c))$ is strictly negative for any $c < C_V$, which implies that it is suboptimal for a financially constrained firm to subsidize the DMM. ■

C.2 Proof of the analysis in Section 4.2.2 (FTT)

Because the FTT is ultimately borne by shareholders, it does not affect the liquidity providers' zero-profit condition directly. However, it does so indirectly through its impact on firm value. That is, $\eta(c)$ solves a condition similar to (13) but subject to $\eta(c) + \omega < \chi$. Firm value satisfies:

$$\begin{aligned} \rho V(c;\eta) = (rc + \mu) V'(c;\eta) + \frac{\sigma^2}{2} V''(c;\eta) + \lambda \sup_f [V(c+f;\eta) - V(c;\eta) - f] \\ + \delta \Big[(1 - \min[\eta(c) + \omega; \chi]) V(c,\eta) - V(c,\eta) \Big] . \end{aligned} \tag{38}$$

In this equation, the optimal refinancing size f replenishes the cash reserves, i.e., $f = C_V - c$ (as in the case with no FTT, see Section 4.1). The term $\min[\eta(c) + \omega; \chi]$ reveals that, all else equal, the presence of the FTT makes liquidity-shocked investors less willing to sell the stock (equivalently, more willing to keep the stock in the face of liquidity shocks). Whenever $\eta(c) < \chi - \omega$, firm value solves the following ODE:

$$(\rho + \delta\kappa + \delta\omega) V(c) = (rc + \mu) V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda [V(C_V) - C_V + c - V(c)] - \gamma. \tag{39}$$

The left-hand side of this expression illustrates that the presence of the FTT further increases the return required by investors by the additional term $\delta\omega$. This additional return reduces firm value and implies that, all else being equal, trading firms extract larger concessions from investors. Whenever $\eta(c) > \chi - \omega$, firm value satisfies equation (17). The ODEs are solved to the same boundary conditions reported in Appendix B.

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Table 1: Benchmark parameters.

Symbol	Description	Value
	Firm	
ρ	Risk-free rate	0.02
r	Return on cash	0.01
μ	Cash flow drift	0.05
μ_+	Post-investment cash flow drift	0.06
σ	Cash flow volatility	0.12
ϕ	Recovery rate in liquidation	0.55
λ	Arrival rate of financing opportunities	0.75
L	Credit line limit	0.08
β	Credit line spread over risk-free	0.015
	Market transactions	
δ	Arrival rate of liquidity shocks	0.7
χ	Loss due to liquidity shocks	0.02
κ	Liquidity providers' funding cost	0.003
γ	Liquidity providers' participation cost	0.007
$\bar{\eta}$	DMM's bid-ask cap	0.008
ω	Financial transaction tax	0.001

Table 2: Effect of bid-ask spreads on corporate outcomes.

The table reports the target cash level, the average probability of external financing, of liquidation, and payout, the zero-NPV investment cost, and firm value (at the target cash level C_V) when varying the bid-ask spread (as reported in the first column).

Bid-ask spread (Basis points)	Target Cash Level	Financing Probability	Liquidation Probability	Payout Probability	Zero-NPV investment cost	Firm value
0	0.546	87.2%	12.8%	24.9%	0.504	2.773
10	0.534	86.9%	13.1%	25.4%	0.487	2.673
20	0.522	86.6%	13.4%	25.9%	0.471	2.580
30	0.510	86.3%	13.7%	26.4%	0.455	2.493
40	0.498	86.0%	14.0%	26.9%	0.441	2.411
50	0.486	85.6%	14.4%	27.5%	0.427	2.334
60	0.473	85.2%	14.8%	28.0%	0.414	2.262
70	0.460	84.8%	15.2%	28.7%	0.401	2.193
80	0.447	84.3%	15.7%	29.3%	0.389	2.128
90	0.432	83.8%	16.2%	30.1%	0.378	2.066
100	0.417	83.2%	16.8%	30.9%	0.366	2.006
120	0.383	81.6%	18.4%	32.9%	0.345	1.895
140	0.340	79.2%	20.8%	35.5%	0.323	1.792
160	0.285	74.7%	25.3%	39.4%	0.303	1.694
200	0.133	42.8%	57.2%	49.9%	0.277	1.510

Table 3: Bid-ask spreads and probability of forced liquidation.

The table reports the firm's probability of liquidation at different levels of cash reserves (i.e., at $C_V/2$, $C_V/4$, and $C_V/8$) when varying the bid-ask spread (as reported in the first column).

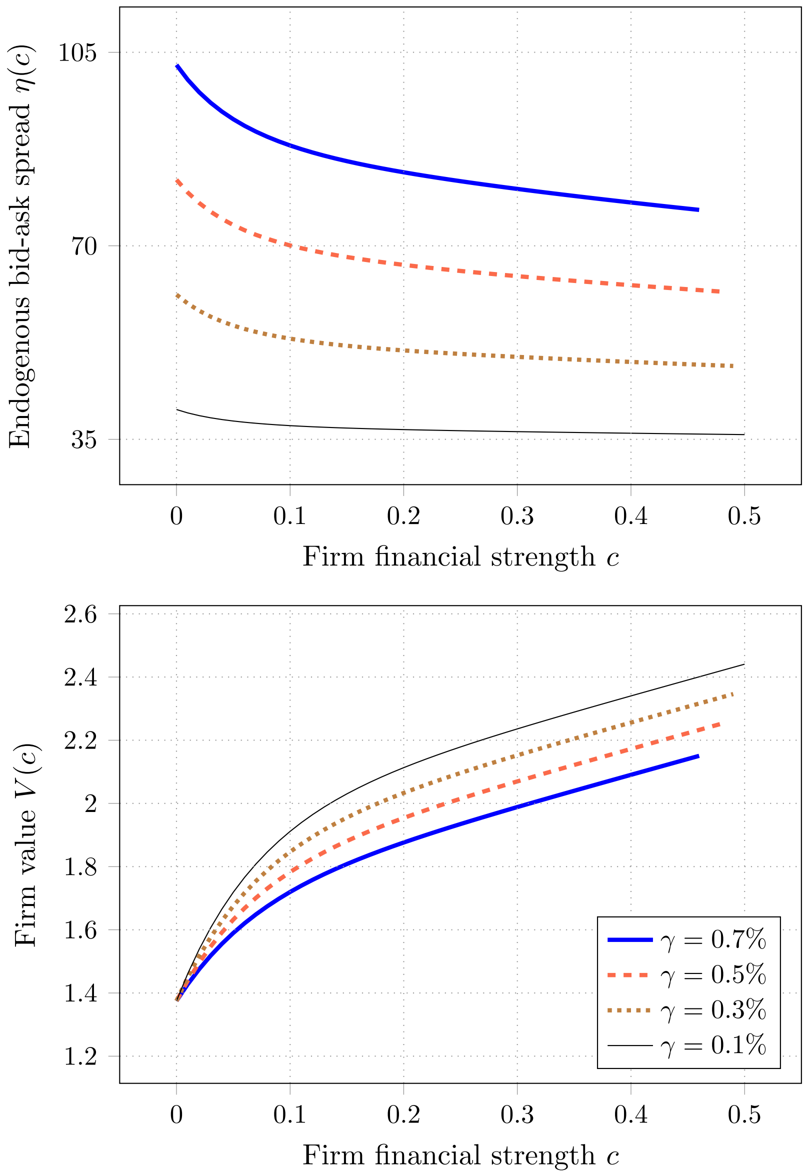
Bid-ask spread (Basis points)	$C_V/2$	$C_V/4$	$C_V/8$
0	1.97%	14.1%	37.6%
10	2.15%	14.7%	38.5%
20	2.34%	15.4%	39.3%
30	2.56%	16.1%	40.1%
40	2.79%	16.8%	41.0%
50	3.06%	17.5%	41.9%
60	3.35%	18.3%	42.9%
70	3.69%	19.2%	43.9%
80	4.08%	20.2%	45.0%
90	4.53%	21.2%	46.1%
100	5.08%	22.5%	47.4%
120	6.58%	25.4%	50.5%
140	9.09%	29.7%	54.5%
160	14.0%	36.6%	60.3%
180	25.3%	48.4%	69.2%
200	51.1%	68.8%	82.3%

Table 4: Liquidity providers' participation frictions and corporate outcomes.

The table reports the change in the target cash level, in the probability of liquidation, in the probability of payout, in the zero-NPV investment cost, and in firm value (calculated at the target cash level) when accounting for participation fees (γ) and funding costs (κ) borne by liquidity providers with respect to a benchmark environment with no trading frictions (in which the bid-ask spread is zero). Each line represents an environment in which γ or κ are varied.

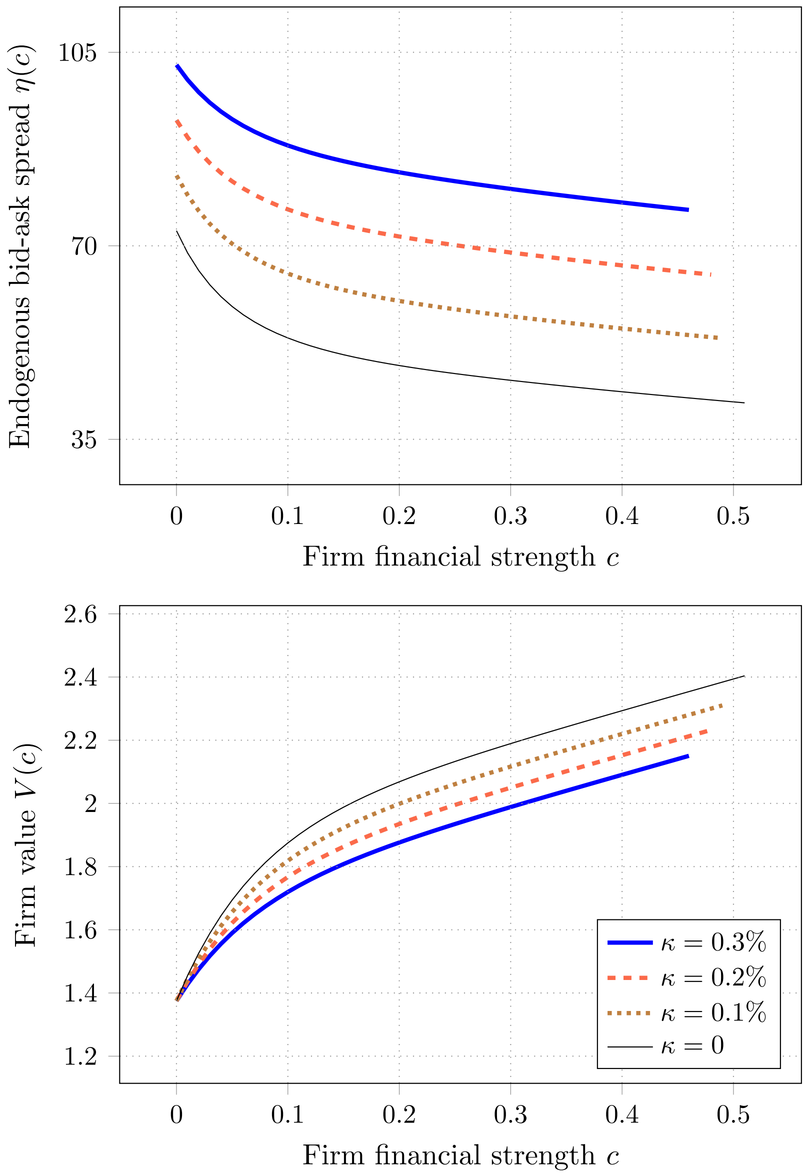
Participation friction	Target cash level ($\Delta\%$)	Liquidation probability (Δ)	Payout probability (Δ)	Zero-NPV investment cost ($\Delta\%$)	Firm value ($\Delta\%$)
$\gamma = 0.1\%$	-7.44%	1.04%	1.67%	-9.86%	-11.80%
$\gamma = 0.3\%$	-9.43%	1.35%	2.14%	-10.19%	-15.24%
$\gamma = 0.5\%$	-11.77%	1.73%	2.73%	-10.63%	-18.71%
$\gamma = 0.7\%$	-14.60%	2.22%	3.46%	-11.20%	-22.23%
$\gamma = 0.9\%$	-18.17%	2.89%	4.42%	-12.03%	-25.81%
$\kappa = 0.1\%$	-8.84%	1.25%	2.00%	-4.54%	-16.42%
$\kappa = 0.2\%$	-11.66%	1.71%	2.70%	-7.95%	-19.41%
$\kappa = 0.3\%$	-14.60%	2.22%	3.46%	-11.20%	-22.23%
$\kappa = 0.4\%$	-17.73%	2.80%	4.30%	-14.35%	-24.88%
$\kappa = 0.5\%$	-21.09%	3.49%	5.25%	-17.41%	-27.40%

Figure 1: Endogenous liquidity provision and firm value when liquidity providers face participation frictions.



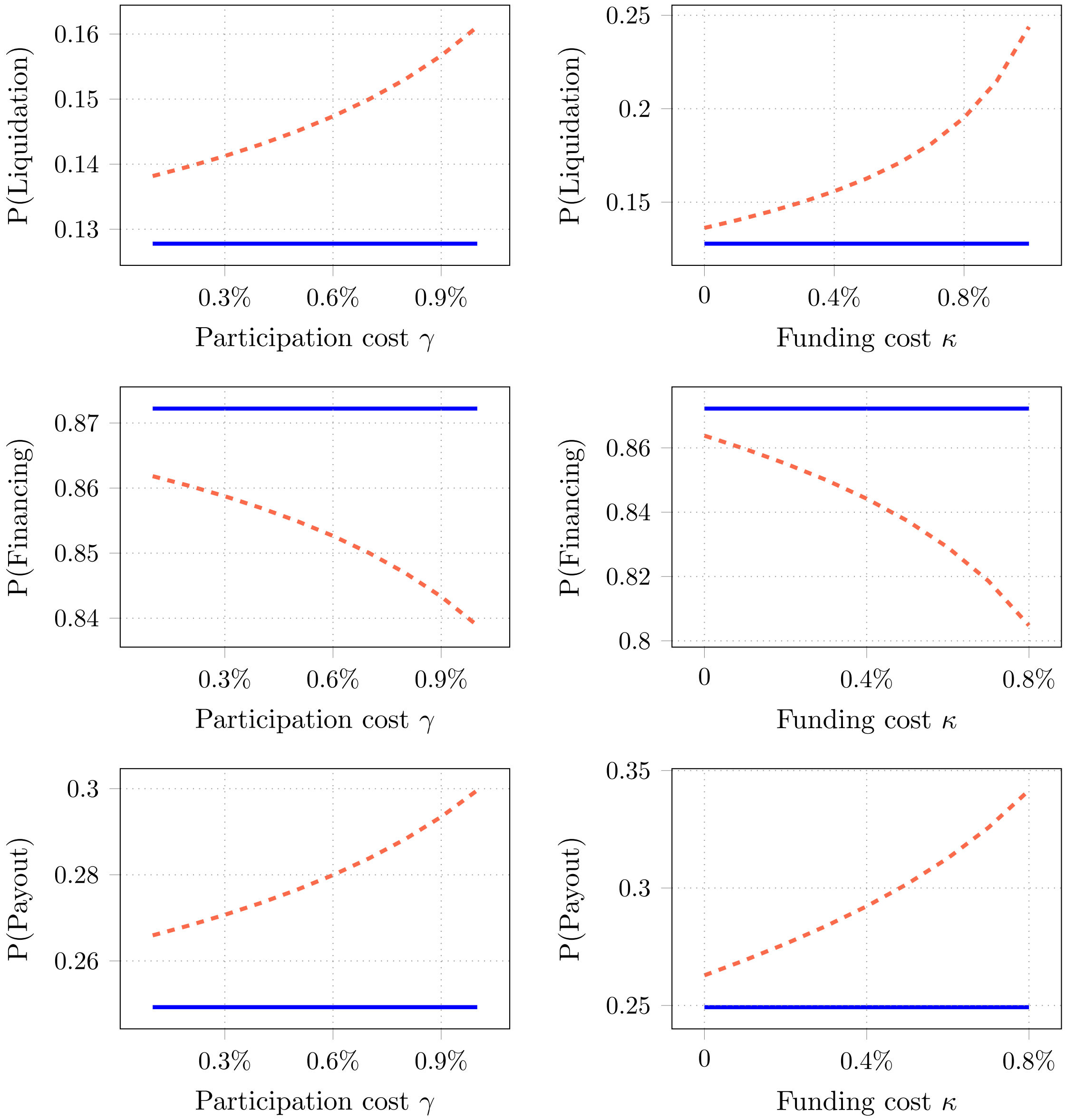
The figure shows the endogenous bid-ask spread (in basis points) as well as firm value as a function of the firm financial strength (i.e., the case reserves c) when varying the participation fee γ borne by liquidity providers.

Figure 2: Endogenous liquidity provision and firm value when liquidity providers face participation frictions.



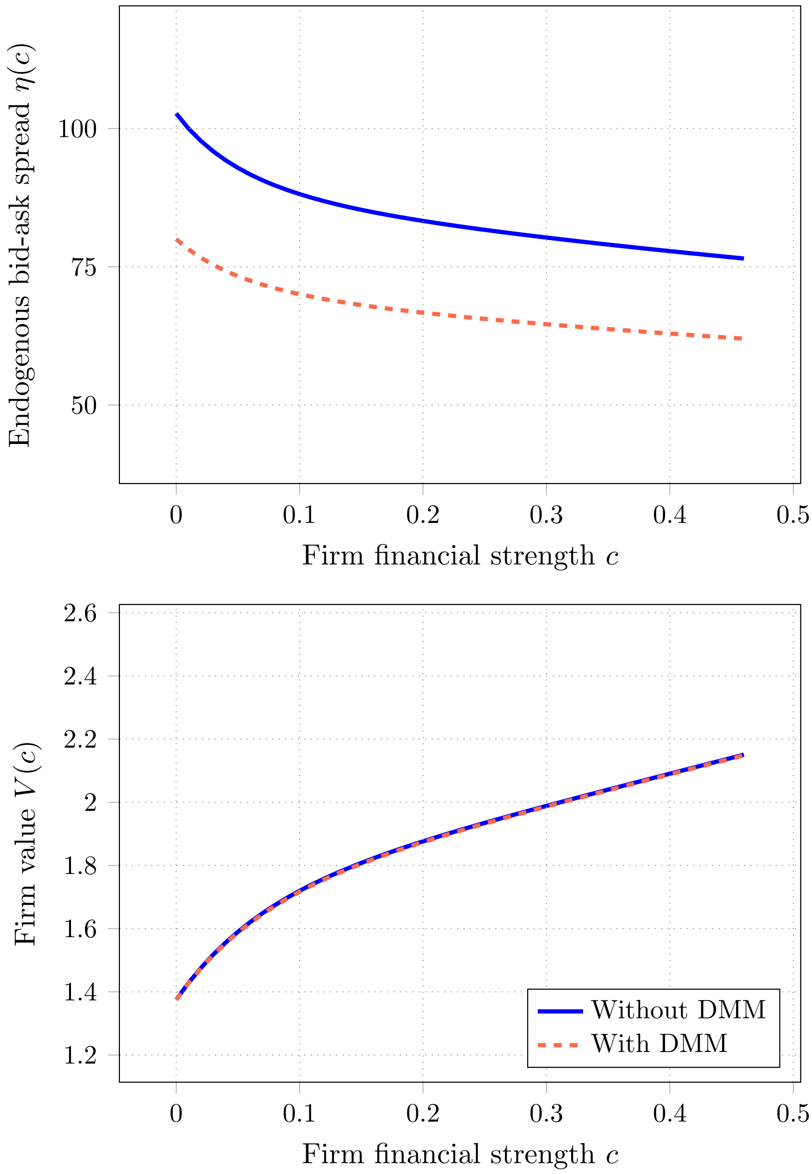
The figure shows the endogenous bid-ask spread (in basis points) as well as firm value as a function of the firm financial strength (i.e., the cash reserves c) when varying the funding cost κ faced by liquidity providers.

Figure 3: The impact of participation frictions on the firm probability of liquidation, financing, and payout.



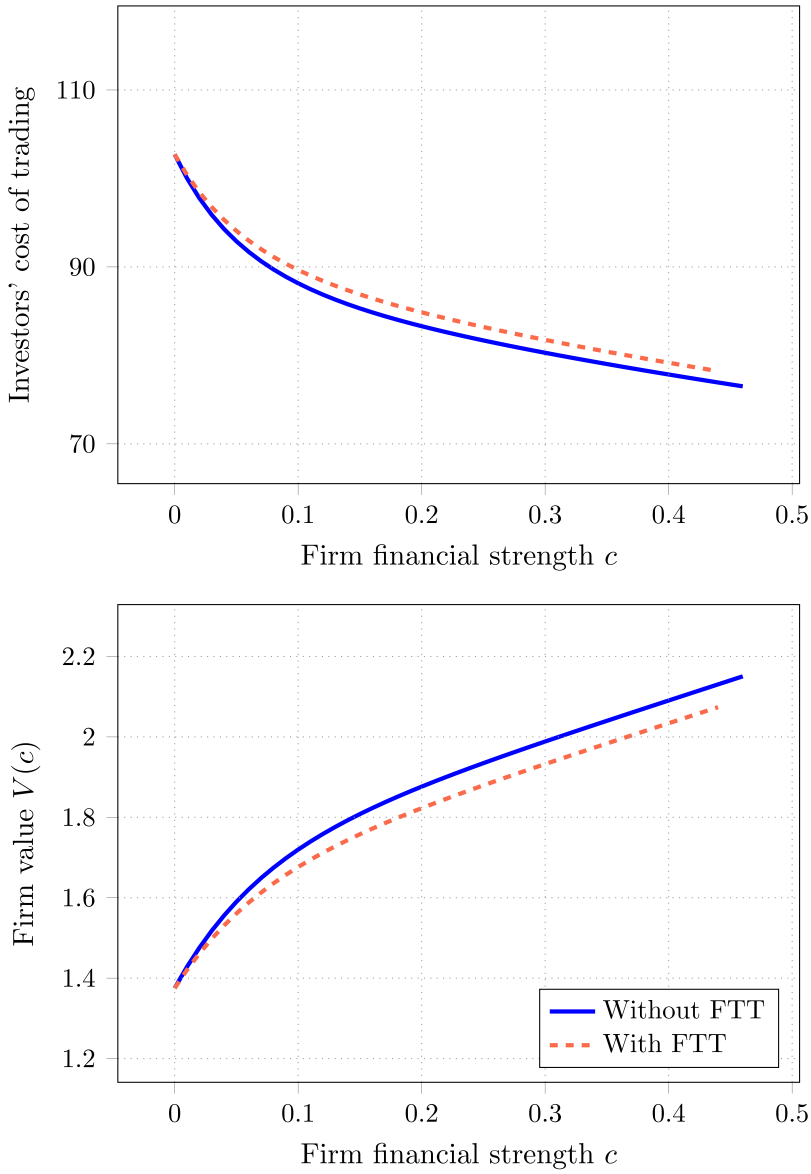
The figure shows the probability of liquidation (first panel), the probability of external financing (middle panel), and the probability of payout (bottom panel) as a function of the participation cost γ (left panel) and the funding cost κ (right panel) borne by liquidity providers. The blue solid line refer to a firm whose shareholders face no bid-ask spread when trading the stock, whereas the red dashed lines refer the case in which bid-ask spreads is endogenously set by liquidity providers.

Figure 4: Firm-funded designated market makers (DMM).



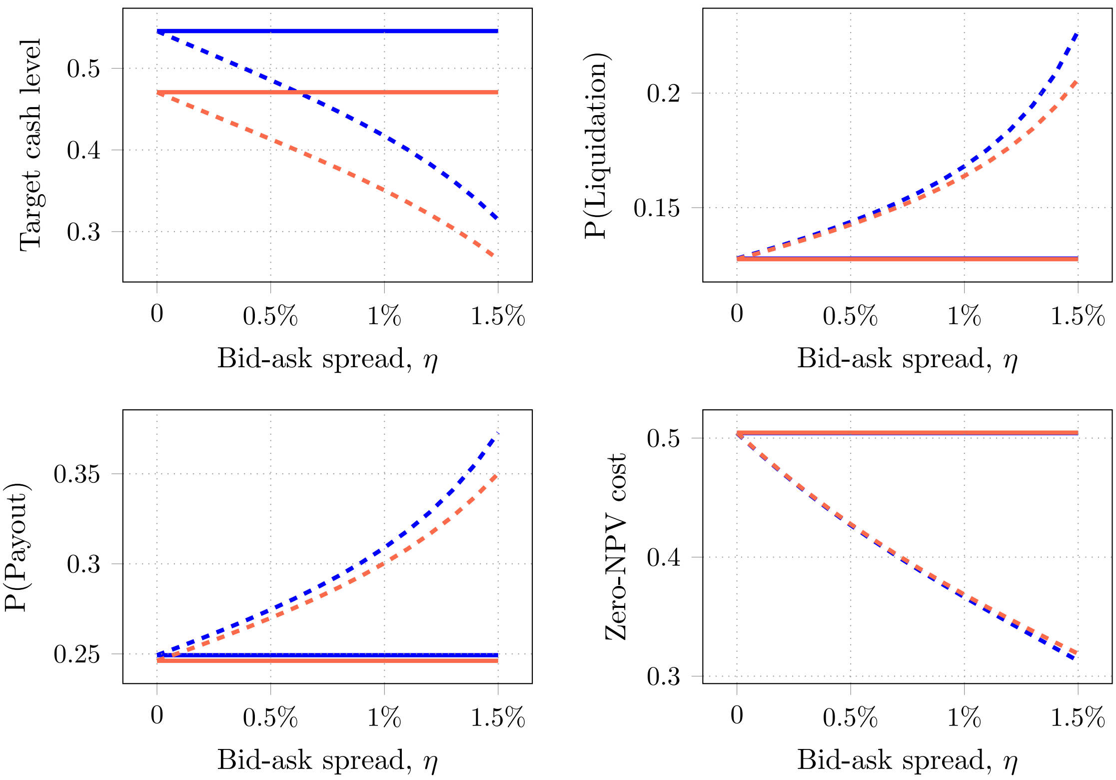
The figure shows the endogenous bid-ask spread (in basis points) as well as firm value as a function of the firm financial strength (cash reserves c). The solid blue line depicts the environment with endogenous bid-ask spread provided by competitive liquidity providers, whereas the dashed red line depicts the environment in which the firm enters the contract with the designated market maker (DMM).

Figure 5: Financial transaction tax (FTT)



The figure shows the bid ask-spread competitively set by liquidity providers (in basis points) as well as firm value as a function of the firm financial strength (the firm cash reserves c). The solid blue line depicts the environment with no financial transaction tax (FTT), whereas the dashed red line depicts the environment in which investors face a FTT.

Figure 6: Trading costs and credit line availability.



The figure shows the target level of cash reserves, the probability of liquidation, the maximum investment cost, and the payout probability, as a function of the bid-ask spread (η). The blue lines refer to a firm with no access to bank credit and when the bid-ask spread is zero (solid line) or positive (dashed line). The red lines refer to a firm having access to bank credit and when the bid-ask spread is zero (solid line) or positive (dashed line).