Supplementary Appendix to "Short-Term Debt and Incentives for Risk-Taking"[*]

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July 8, 2019

SA.1. Robustness to alternative model specifications

SA.1.1. Increasing debt exposure via credit lines

In our benchmark analysis, the firm is forced into liquidation when cash reserves are depleted and access to the equity market is prohibitively expensive. We now assess the robustness of our main results by allowing the firm to take on additional debt via credit line drawdowns (see also Bolton, Chen, and Wang, 2011; Pierre, Villeneuve, and Warin, 2016). In practice, credit lines provide firms with immediate liquidity that can be used in times of need (see Sufi, 2009). In our model, they allow the firm to acquire flexibility in their debt and liquidity policies, with a total amount of (net) debt varying between $P - W^*$ and $P + L$, where L is the pre-established limit on the credit line.

Specifically, assume that the firm has access to a credit line with pre-determined limit $L \geq 0$. For the amount of credit that the firm uses, the interest spread over the risk-free rate is $\beta > 0$. Because of this spread, the firm will optimally avoid using its credit line before exhausting internal funds. That is, the firm uses cash as the marginal source of financing if $w \in [0, W^*(L)]$ (the cash region), where $W^*(L)$ denotes the target cash level when the firm has access to a credit line. Conversely, the firm draws funds from the credit line when $w \in [-L, 0]$ (the credit line region). In the following, we assume that the credit line has priority over short-term debt and that $L < \ell$, implying that the credit line is fully collateralized.

We derive the system of equations for the values of equity and short-term debt in the presence of a credit line, following the analysis in the main text. The firm uses cash as the marginal source of financing if $w \in [0, W^*(L)]$ (the cash region), where $W^*(L)$ denotes the target cash level as a function of L. Conversely, the firm draws funds from the credit line when $w \in [-L, 0]$ (the credit line region). In the cash region, the value

of equity satisfies the following equation:

$$rE(w) = [(1-\theta)((r-\lambda)w+\mu-C)+m((1-\kappa)D(w)-P)]E'(w)+\frac{1}{2}\left((1-\theta)\sigma\right)^2 E''(w).$$

(SA1)

In the credit line region, the firm needs to pay interests on borrowed funds, and the value of equity satisfies

$$rE(w) = [(1-\theta)((r+\beta)w+\mu-C)+m((1-\kappa)D(w)-P)]E'(w)+\frac{1}{2}\left((1-\theta)\sigma\right)^2 E''(w).$$

(SA2)

Similarly, the value of short-term debt satisfies the following ODE

$$(r+m)D(w) = [(1-\theta)((r-\lambda)w+\mu-C)+m((1-\kappa)D(w)-P)]D'(w) \\ +\frac{1}{2}\left((1-\theta)\sigma\right)^2 D''(w)+C+mP$$

(SA3)

in the cash region, whereas it satisfies the following ODE

$$(r+m)D(w) = [(1-\theta)((r+\beta)w+\mu-C)+m((1-\kappa)D(w)-P)]D'(w) \\ +\frac{1}{2}\left((1-\theta)\sigma\right)^2 D''(w)+C+mP$$

(SA4)

in the credit line region.

The system of equations is solved subject to the following boundary conditions at the liquidation boundary $(-L)$ and at the payout boundary $(W^*(L))$:

$$E(-L) = E'(W^*(L)) - 1 = E''(W^*(L)) = 0,$$

(SA5)

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and

$$D(-L) - (\ell - L) = D'(W^*(L)) = 0, \tag{SA6}$$

as well as the continuity and smoothness conditions where the credit line and cash regions are pieced together

$$\lim_{w\uparrow 0} E(w) = \lim_{w\downarrow 0} E(w), \text{ and } \lim_{w\uparrow 0} E'(w) = \lim_{w\downarrow 0} E'(w) \tag{SA7}$$

$$\lim_{w\uparrow 0} D(w) = \lim_{w\downarrow 0} D(w), \text{ and } \lim_{w\uparrow 0} D'(w) = \lim_{w\downarrow 0} D'(w). \tag{SA8}$$

Fig. SA.1 describes the effects of credit lines on the values of corporate securities and rollover imbalances.

Insert Fig. SA.1 Here

Because a credit line serves as an additional source of liquidity, the figure shows that credit lines reduce the need for large cash balances in that the target cash level is smaller when $L > 0$ (see also Bolton, Chen, and Wang, 2011; or Décamps, Gryglewicz, Morellec, and Villeneuve, 2017). By reducing the expected cost of financing frictions, credit lines increase the values of debt and equity in the cash region. Nonetheless, credit lines reduce the value of short-term debt in the credit line region. The reason is that the credit line has to be paid in full before debtholders can collect any liquidation proceeds. The resulting lower payoff to short-term debt in liquidation leads to larger rollover losses when the firm is close to exhausting the credit line (see the bottom right panel). This implies that *senior* credit lines strengthen the amplification mechanism highlighted in the main text and, therefore, shareholders' incentives for risk-taking. This analysis therefore not only confirms our results on the effects of short-term debt on risk-taking incentives, but also

shows that these effects can be magnified by the presence of a secured credit line.

SA.1.2. Introducing financing frictions in a Leland-type setup

Our result that short-term debt generates risk-taking incentives goes against the long-standing idea that short-term debt reduces the agency cost of asset substitution, as discussed for example in Leland and Toft (1996) or Leland (1994b, 1998). In this section, we show that this result is not driven by the assumption about the stochastic process governing the firm's cash flows but rather by the fact that financing frictions constrain shareholders' default decision.[1] To do so, we consider in this section a setup à la Leland (1994b, 1998) in which we relax the assumption that shareholders have deep pockets and can choose the timing of default that maximizes equity value. The results derived in this section also hold in a Leland and Toft (1996) setup, in which bond expirations are uniformly spread out over time.

Consider a firm whose unlevered asset value $V = (V_t)_{t \geq 0}$ follows a geometric Brownian motion:

$$dV_t = (\mu - \delta)V_t \, dt + \sigma V_t \, dZ_t, \tag{SA9}$$

where μ is the total expected rate of return, δ is the constant payout rate, and dZ_t is the increment of a standard Brownian motion. The firm is financed with equity and short-term debt, as described in Section 2 of the main text. In Leland (1994b, 1998) or Leland and Toft (1996), the process in Eq. (SA9) continues without time limit unless V falls to a default-triggering value V_B, which is endogenously determined to maximize equity

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[1]In line with previous dynamic models with financing frictions such as Décamps, Mariotti, Rochet, and Villeneuve (2011) or Hugonnier, Malamud and Morellec (2015), cash flows are governed by an arithmetic Brownian motion in our model. In Leland-type models, total cash flows and asset values are governed by a geometric Brownian motion.

value. In these models, shareholders can inject funds in the firm instantaneously and at no cost, and equity value is a convex function of asset value when debt maturity is infinite or sufficiently long. Shareholders thus have incentives to increase asset risk, which is detrimental to debtholders. Conversely, if maturity is sufficiently short, increasing risk does not benefit shareholders, except when default is imminent. In this setting, short-term debt acts as a disciplining device by decreasing shareholders' risk-taking incentives.

Suppose now that shareholders *cannot* optimize the timing of default because of financing frictions, debt covenants, or regulatory constraints, and denote by V_b the exogenous threshold (for asset value) triggering default. To consider relevant cases, we assume that V_b is greater than V_B, so that shareholders are forced to liquidate the firm's assets early, when suboptimal for them. Default can be interpreted in this context as being triggered by the breach of a net-worth covenant. Alternatively, it can be interpreted as a liquidity default caused by financing frictions. As shown by Leland (1994a) and Toft and Prucyk (1997), when the default boundary is exogenous and sufficiently large, equity value becomes a concave function of asset value. In this case, shareholders are effectively risk-averse and have no risk-taking incentives.[2]

We now show that short-term debt can restore the convexity of equity value when the default boundary is exogenous. The mechanism is similar to that analyzed in Section 3.2 of the paper. To see this, consider the expected net cash flow to shareholders when varying debt maturity M. When $M = \infty$, this expected net cash flow is given by

$$[\, \delta V_t - (1 - \theta)C\,]dt, \tag{SA10}$$

on any interval of length dt, which is the total firm payout minus the after-tax coupon

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[2] A similar result obtains in the models of cash management with financing frictions and infinite maturity debt developed by Bolton, Chen, and Wang (2015) or Hugonnier and Morellec (2017).

payment. When M is finite, the net cash flow to equityholders is given by

$$[\,\delta V_t - (1 - \theta)C + \underbrace{m((1 - \kappa)D(V_t; m) - S)}_{\text{Rollover imbalance}}\,]dt \qquad \text{(SA11)}$$

where the last term in the square bracket represents the firm's rollover imbalance. When asset value is low so that $(1 - \kappa)D(V_t; m) < S$, the firm faces rollover losses. When average debt maturity is shorter, the fraction m of debt that needs to be rolled over each time interval is larger, which magnifies rollover losses as fundamentals deteriorate. For sufficiently short debt maturity, Eq. (SA11) can be negative even for values of V_t that make Eq. (SA10) positive. That is, if rollover losses are sufficiently large, expected net cash flows to shareholders turn negative. In this case, shareholders hold an out-of-the-money option and have incentives to increase asset risk.

Insert Fig. SA.2 Here

Fig. SA.2 provides an illustration of this result by plotting the value of equity and the marginal value of equity as functions of the value of the firm's assets, for different debt maturities. In this figure, we base our parametrization on Leland (1994b) and set the risk-free rate to 7.5%, the cash flow volatility to 0.20, bankruptcy costs to 0.5, the tax rate to 0.35, the payout rate to 0.07, and the debt issuance cost to 0. Additionally, we set the value of debt principal and coupon to 65 and 6, respectively. In the top panel, we impose an exogenous default boundary equal to $V_b = 90$, which is larger than the endogenous default boundaries. The top panels of the figure demonstrate that incentives for risk taking are less pronounced for long-term debt than for short-term debt *if* shareholders are constrained in their default decisions. If debt maturity is sufficiently long, equity value is concave for all asset values and equityholders have no risk-taking

incentives. If debt maturity is short, equity value becomes convex when asset value is sufficiently close to the default threshold. As shown in the bottom panels, this is not the case when shareholders are unconstrained in their default decisions. In this case, long-term debt is associated with risk-taking incentives for any V.

It is worth noting that risk-taking incentives in this context arise in fundamentally solvent firms, i.e. firms for which Eq. (SA10) is positive. They are driven by liquidity problems rather than by solvency problems. Because financing frictions are key to this mechanism and because they lead shareholders to value retained earnings, our baseline model is one in which we allow firms to keep cash reserves, as in Décamps, Mariotti, Rochet, and Villeneuve (2011) or Bolton, Chen, and Wang (2011).

SA.1.3. Time-varying profitability/solvency

In the previous subsection, we have relaxed the assumption that expected firm profitability is constant by casting our research question in a Leland-type framework. We now follow a different approach; specifically, we introduce time-varying firm profitability in the setup analyzed in Section 5.1 of the main text. In this extension, we assume that firm profitability/solvency is state-contingent. We do so by assuming that the cash flow drift is state dependent and denoted by μ_G in the good state and by μ_B in the bad state. We assume that $\mu_G > \mu_B$ to capture the notion that profitability decreases in the bad state B (i.e., when fresh financing is unavailable to the firm). All the other assumptions are as in Section 5.1 of the paper.

Under these assumptions, the valuation equation of equity in any state i satisfies the

following equation:

$$rE_i(w) = [(1-\theta)((r-\lambda)w + \mu_i - C) + m((1-\kappa)D_i(w) - P)]E_i'(w)$$
$$+ \frac{1}{2}((1-\theta)\sigma)^2 E_i''(w) + \pi_i[E_j(w) - E_i(w)] \tag{SA12}$$

in the earnings retention region. This equation is subject to the same boundary conditions as in Section 5.1. Similarly, the valuation equation for debt in state i is given by

$$(r+m)D_i(w) = [(1-\theta)((r-\lambda)w + \mu_i - C) + m((1-\kappa)D_i(w) - P)]D_i'(w)$$
$$+ \frac{1}{2}((1-\theta)\sigma)^2 D_i''(w) + C + mP + \pi_i[D_j(w) - D_i(w)] \tag{SA13}$$

in the earnings retention region, which is subject to the same boundary conditions as in Section 5.1.

Insert Fig. SA.3 Here

Fig. SA.3 shows the rollover imbalance, the value of equity, and the marginal value of cash from the shareholder's perspective in the good and in the bad state when assuming that profitability is $\mu_G = 0.10$ in the good state and $\mu_B = 0.06$ in the bad state. Given that the long-run probability of being in state G is given by $\frac{\pi_B}{\pi_B+\pi_G}$, these values imply an average profitability of 0.09 as in the baseline calibration of the model. As in the baseline version of the model in which expected profitability is constant, the figure shows that the value of equity can be locally convex when rollover losses are large, in which case shareholders have incentives for risk-taking. Conversely, incentives for risk-taking do not arise for firms that are financed with infinite maturity debt (or, more generally, when debt maturity is sufficiently long), in which case equity is always concave. Overall,

Fig. SA.3 demonstrates that short-term debt still generates incentives for risk-taking in the case in which firm profitability/solvency are time-varying—that is, our main result is not specific to the assumption that expected profitability is constant.

SA.2. Implications for capital structure

In this section, we investigate the capital structure implications of our economic mechanism. As in Leland (1994), the coupon is chosen at the initial date to maximize the sum of equity and debt values (as calculated in Section 3.1 of the main text), under the constraint that debt is issued at par. We use different initial levels of cash reserves to account for varying degrees of financial constraints at the time the firm is set up. That is, shareholders solve the problem

$$V(w_0; C, m, P) \equiv \sup_{C \in \mathbb{R}_+} \left[E(w_0; C, m, P) + D(w_0; C, m, P) \right], \tag{SA14}$$

under the constraint that short-term debt is initially issued at par $D(w_0; C, m, P) = P$, and under the budget constraint $w_0 = W_{0-} + (1-\kappa)P - I$, where I is the initial investment cost and W_{0-} is the initial cash endowment that constrained shareholders are able to inject into the firm. The budget constraint implies that the firm cannot freely raise the optimal cash amount at time zero, reflecting financing constraints and consistent with our key assumption that shareholders do not have deep pockets.

Table SA.1 shows the capital structure that maximizes firm value as a function of debt maturity. When debt maturity is infinite, there are no rollover imbalances. In this case, the optimal debt level balances the tax benefits of debt with bankruptcy costs. When debt maturity is finite, two additional factors shape capital structure choices. First, a short debt maturity imposes larger rollover losses when cash reserves are low, which increases the cost of debt and decreases the firm's debt capacity. Second, a short

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debt maturity increases the proceeds from debt rollover when cash reserves are large (and default risk is low), which decreases the cost of debt and increases the firm's debt capacity. When debt maturity is relatively short, the first effect dominates and the threat of large rollover losses makes the coupon that maximizes firm value smaller compared to the infinite maturity case. That is, by generating substantial rollover losses, a shorter maturity decreases the firm's debt capacity and optimal leverage. The second effect may dominate when debt maturity is finite and relatively long; i.e., the lower cost of debt leads the firm to increase the optimal coupon compared to the infinite maturity case.

Insert Table SA.1 Here

In our model, a decrease in average debt maturity decreases the value of risky debt by increasing rollover losses and default risk. As mentioned earlier, debtholders suffer from the downside risk and do not capture any upside from issuing short-term. Therefore, the value of debt is the largest when maturity is infinite. This is what we call the *debt effect*. For shareholders, however, a shortening of average debt maturity has contrasting effects depending on the firm's cash reserves (see Figure 3 of the paper). When cash reserves are low, a shorter debt maturity leads to larger rollover losses, which decrease the value of equity. When cash reserves are large, a shorter maturity leads to larger net proceeds from rolling over maturing debt, which increase the value of equity. This is what we call the *equity effect*. Consistent with this contrasting effects, Table SA.1 shows that firm value is non-monotonic with debt maturity if the initial level of cash reserves is sufficiently large—$W^*/2$ in our numerical example. The underlying motive for choosing short-term debt maturities in our model is thus very different from previous contributions, in which short-term debt maturity allows firms to *reduce* the agency costs of risk-shifting (Leland and Toft (1996) or Cheng and Milbradt (2012)) or to reduce the

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cost of bond illiquidity (He and Xiong (2012) or He and Milbradt (2014)). In our model, short-term debt maturity decreases the cost of debt for solvent firms, but this benefit needs to be weighted against severe rollover losses when cash flows deteriorate.

While financing frictions are a key ingredient of our analysis, it is also interesting to analyze capital structure choices when shareholders are able to start operations with the optimal amount of cash. That is, at time zero, shareholders need to finance the amount $I + w_0$, i.e., the investment in risky assets (the first term) and in cash (the second term). They do so by raising debt—the associated proceeds are $(1 - \kappa)P$, subject to the constraint that debt is issued at par at time zero, i.e. $D(w_0) = P$—and by injecting their own savings into the firm, W_{0-}. When shareholders can decide how much to inject into the firm, they maximize:

$$\sup_{(C, w_0)} E(w_0; C, m, P) - W_{0-} \tag{SA15}$$

subject to the constraint that debt is issued at par, $D(w_0; C, m, P) = P$, and the budget constraint $W_{0-} + (1 - \kappa)P = w_0 + I$. As our analysis shows, equity and debt values are increasing in cash reserves, so it is straightforward to show that it is optimal for shareholders to start with the target level of cash reserves.[3]

$$\boxed{\text{Insert Table SA.2 Here}}$$

In line with Table SA.1, Table SA.2 shows that when debt maturity is relatively short, the threat of large rollover losses makes the optimal coupon and debt principal smaller compared to the infinite maturity case. When maturity is sufficiently long, the firm

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[3]The marginal value of one dollar paid out to shareholders is one. The marginal value of one dollar held inside the firm is $E'(w)$. Because $E'(w) = 1$ for any $w \geq W^*$, the proceeds from debt issuance exceeding $W^* + I$ are paid out to shareholders. That is, when $(1 - \kappa)P > W^* + I$, shareholders do not need to inject their own funds into the firm at time zero but instead receive the surplus $(1-\kappa)P - W^* - I$ from the debt issue net of the investment in real and financial assets.

increases the optimal coupon and principal compared to the infinite maturity case. Table SA.2 shows that the highest optimal coupon and principal are associated with $M = 30$. Table SA.2 also shows that firm value is maximal for this finite maturity—notably, firm value is non-monotonic in M. Interestingly, the value of equity is the highest when debt maturity is the shortest, i.e., when $M = 1$. However, the cost of injecting funds into the firm (especially when maturity is short, which leads the firm to pick a smaller principal value) implies that shareholders' payoff at time zero is higher if debt maturity is longer. In fact, when maturity is sufficiently long, debt proceeds are enough to cover the initial investment in cash and illiquid assets, so that shareholders may receive a lumpy dividend rather than having to inject funds into the firm.

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Table SA.1. Constrained value-maximizing capital structure. The table reports the value-maximizing capital structure (coupon, principal, and leverage ratio) and initial firm value when varying the average maturity of corporate debt, under the assumption that shareholders cannot freely decide the initial optimal level of cash reserves because of financial constraints.

Maturity (M)	Coupon (C)	Principal (S)	Leverage ratio	Firm Value
AT PAR AT $W^*/3$				
1	n.a.	n.a.	n.a.	n.a.
2	0.022	0.595	33.7%	1.768
3	0.027	0.681	37.8%	1.803
5	0.034	0.808	43.8%	1.844
10	0.043	1.001	52.8%	1.896
30	0.053	1.267	64.8%	1.954
50	0.053	1.306	66.5%	1.964
Inf	0.051	1.318	66.8%	1.974
AT PAR AT $W^*/2$				
1	n.a.	n.a.	n.a.	n.a.
2	0.027	0.747	39.6%	1.887
3	0.032	0.850	43.9%	1.938
5	0.039	1.008	50.4%	1.999
10	0.050	1.267	60.9%	2.082
30	0.064	1.629	75.1%	2.171
50	0.063	1.629	75.0%	2.174
Inf	0.058	1.549	71.5%	2.165

Table SA.2. Optimal capital structure. The table reports the optimal capital structure (coupon, principal), the optimal level of cash reserves, the value of equity, the payoff to shareholders at the outset, the leverage ratio, as well as firm value when varying the average maturity of corporate debt.

| Maturity (M)	Coupon (C)	Principal (P)	Target cash	Equity value	Net equity value	Leverage ratio	Firm value
1	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
2	0.023	0.660	0.283	1.386	0.755	32.3%	2.045
3	0.028	0.808	0.317	1.316	0.798	38.0%	2.123
5	0.035	1.006	0.358	1.215	0.854	45.3%	2.221
10	0.047	1.334	0.415	1.030	0.935	56.4%	2.364
20	0.060	1.696	0.464	0.802	1.017	67.9%	2.498
30	0.067	1.850	0.462	0.681	1.051	73.1%	2.532
50	0.067	1.829	0.397	0.652	1.066	73.7%	2.482
100	0.065	1.765	0.356	0.677	1.068	72.3%	2.442
Inf	0.063	1.703	0.328	0.712	1.069	70.5%	2.415
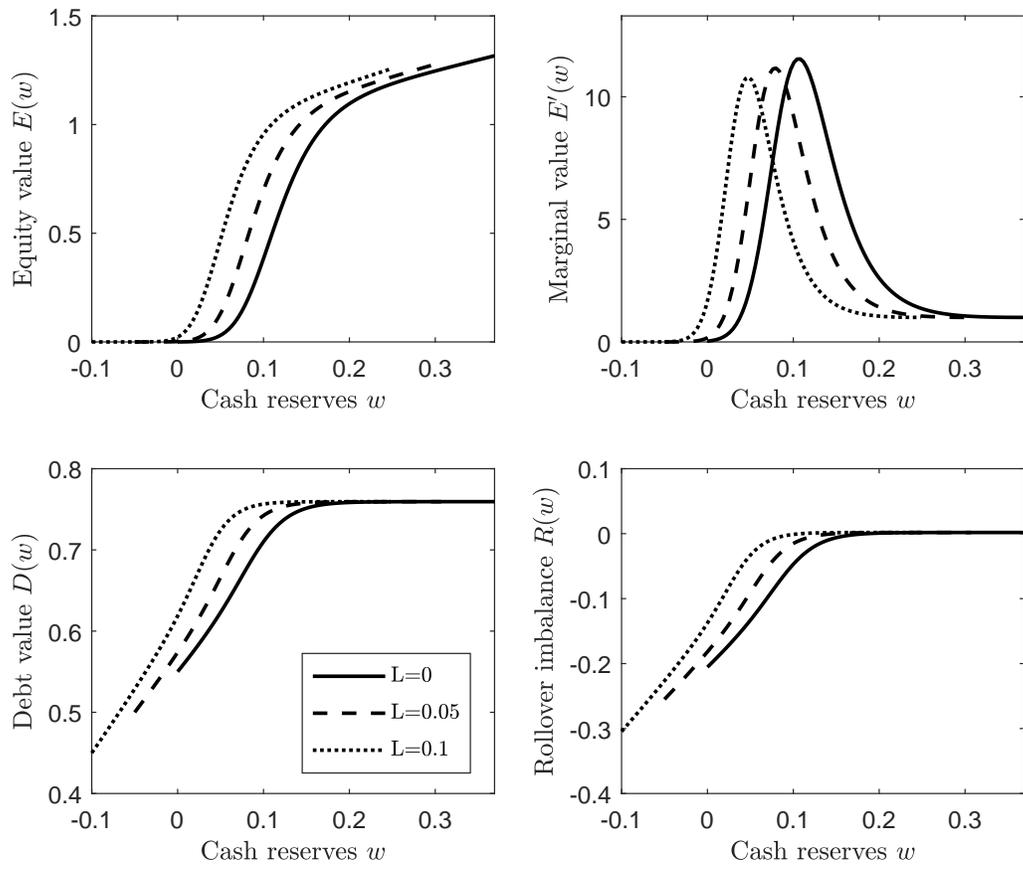
Fig. SA.1. Credit line. The figure plots the value of equity, the marginal value of equity, the aggregate value of short-term debt, and the rollover imbalance in the absence (solid line) and in the presence of credit line availability (the dashed line represents the case $L = 0.05$, whereas the dotted line represents the case $L = 0.1$).
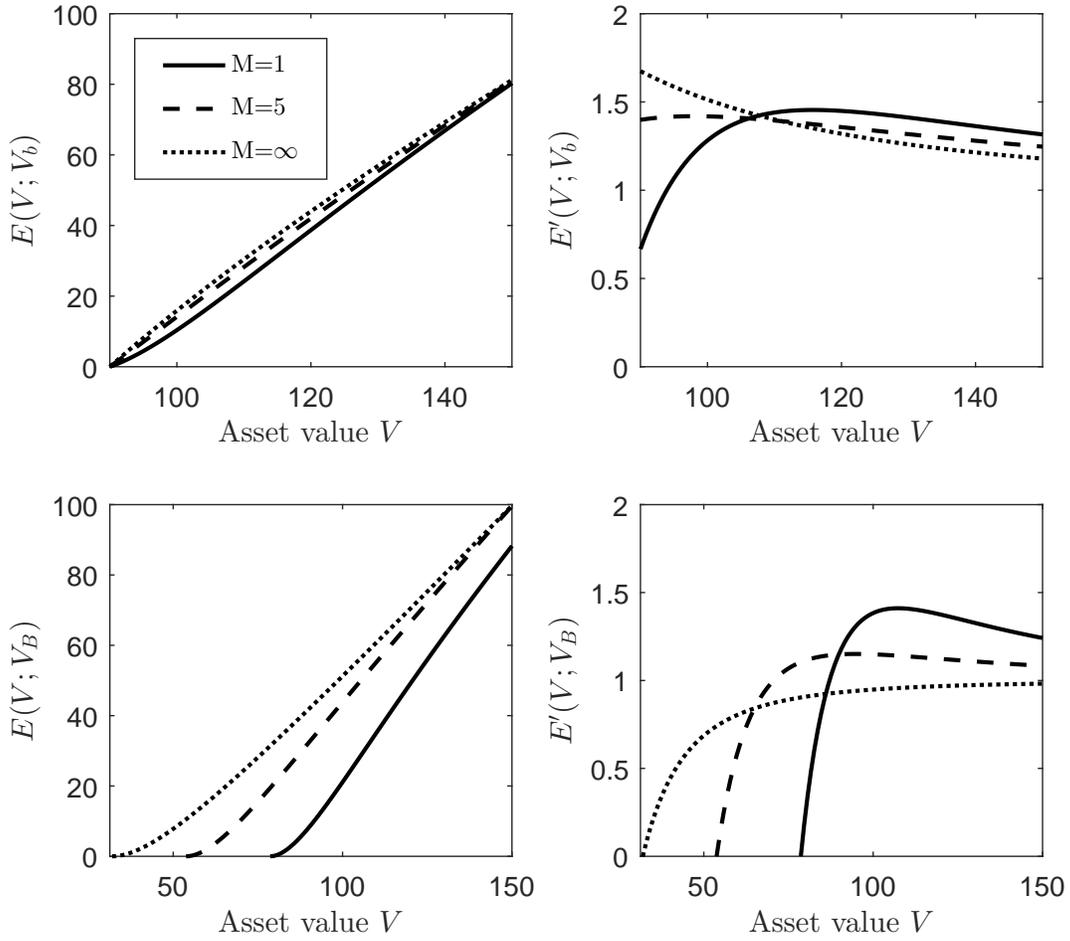
Fig. SA.2. Leland Setup. The figure plots the value of equity $E(V)$ and its sensitivity to asset value $E'(V)$ when the default threshold is exogenous (top panel) or endogenously chosen to maximize equity value (bottom panel), for average debt maturities M of 1 year (solid line), 5 years (dashed line), and infinite (dotted line).
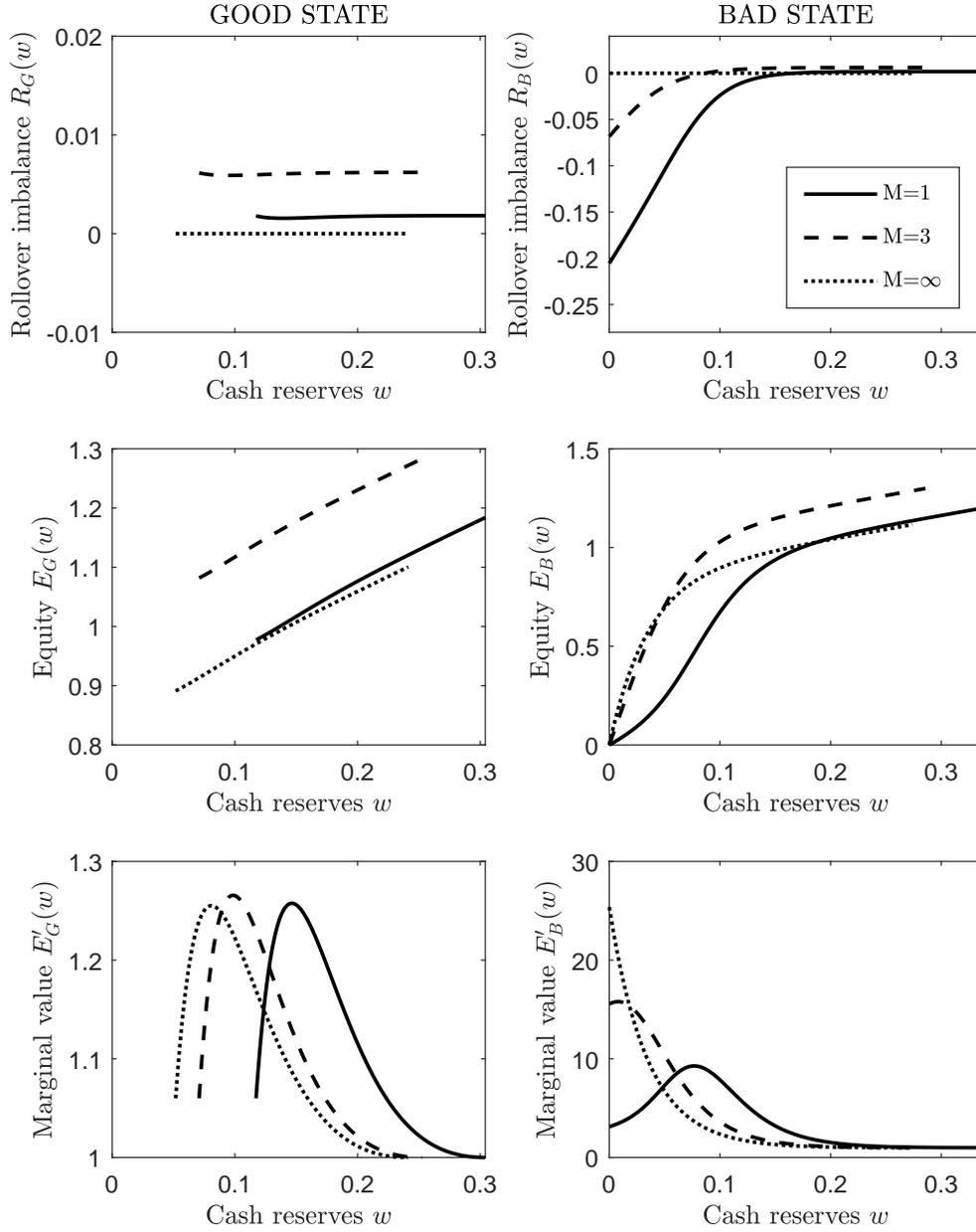
Fig. SA.3. Time-varying firm profitability. The figure plots the rollover imbalance $R_i(w)$, the value of equity $E_i(w)$, and the marginal value of cash for shareholders $E_i'(w)$ as a function of cash reserves $w \in [0, W_i^*]$ in the good state (left panel) and in the bad state (right panel) for average debt maturities M of 1 year (solid line), 3 years (dashed line), and infinite (dotted line). The profitability parameter is $\mu_G = 0.10$ in the good state whereas it is equal to $\mu_B = 0.06$ in the bad state.